MATHEMATICAL WRINKLES

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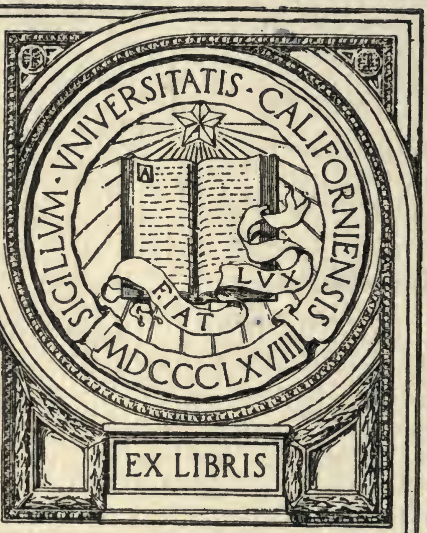
MATHEMATICS

S. I. JONES

GIFT OF
JULIUS WANGENHEIM '87



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MATHEMATICAL WRINKLES

FOR TEACHERS AND PRIVATE LEARNERS

UNIV. OF CALIFORNIA

CONSISTING OF

KNOTTY PROBLEMS; MATHEMATICAL RECREATIONS
ANSWERS AND SOLUTIONS; RULES OF MENSURATION; SHORT METHODS; HELPS, TABLES, ETC.

BY

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AND LITERARY COLLEGE, GUNTER, TEXAS

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Gift of J. Wangenheim

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"ALBERT SMITH, in one of his amusing novels, describes a woman who was convinced that she suffered from 'cobwigs on the brain.' This may be a very rare complaint, but in a more metaphorical sense, many of us are very apt to suffer from mental cobwebs, and there is nothing equal to the solving of puzzles and problems for sweeping them away. They keep the brain alert, stimulate the imagination, and develop the reasoning faculties. And not only are they useful in this indirect way, but they often directly help us by teaching us some little tricks and 'wrinkles' that can be applied in the affairs of life at the most unexpected times, and in the most unexpected ways."

H. E. DUDENEY.

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PREFACE

The following pages contain many mathematical problems, puzzles, and amusements of past and present times. They have a long and interesting history and are part of the inheritance of the school.

This book is intended to be a helpful companion to teachers, and to impart to students a knowledge of the application of mathematical principles, which cannot be obtained from textbooks.

The present-day teacher has little time for the selection of suitable problems for supplementary work. This book is designed to meet the requirements of teachers who feel such extra assignments essential to thorough work. Whatever text is used, the necessity for a work of this kind is felt from the fact that fresh problems produce interest and stimulate investigation.

Originality is not claimed for all of the problems, but for many of them. They have been compiled from various sources. The author's aim has been to select problems not only instructive, but also interesting and amusing.

The rules of Mensuration and Short Methods have been included because of their usefulness. On account of the various helps placed in this book, it will serve as a handbook of mathematics to both teachers and pupils.

The solutions to only part of the problems are given. In some cases solutions of considerable length are given, but at other times only the answers are given. Had the full solutions and proofs been given in every case, either half the problems would have had to be omitted, or the size of the book greatly increased.

The author acknowledges his indebtedness to many friends for helpful suggestions. Specially is he under obligation to the late Dr. G. B. M. Zerr, Philadelphia, Pa., for critically reading the manuscript. A few of his solutions published in the leading Mathematical Journals have been used on account of their beauty and simplicity. He is indebted to Dr. H. Y. Benedict and Mr. J. W. Calhoun, of the University of Texas, for reading the manuscript and offering many valuable suggestions and criticisms. He is very thankful to Dr. George Bruce Halsted, head of the department of mathematics of the Colorado State Teachers' College at Greeley, for criticising the Definitions, Historical Notes, and Classifications. He is also specially indebted to Professor Dow Martin, of the Biblical and Literary College of Gunter, Texas, for reading and correcting the proof-sheets.

Any correction or suggestion relating to these problems and solutions will be most thankfully received.

It is hoped that this small volume may produce higher and more noble results in awakening a real love and interest among the great body of teachers and students for the study of mathematics, "the oldest and the noblest, the grandest and the most profound, of all sciences."

SAMUEL I. JONES.

GUNTER, TEXAS.

MATHEMATICAL WRINKLES

ARITHMETICAL PROBLEMS

1.* Between 3 and 4 o'clock I looked at my watch and noticed the minute hand between 5 and 6; within two hours I looked again and found that the hour and minute hands had exchanged places. What time was it when I looked the second time?

2.* A tree 120 feet high was broken in a storm, so that the top struck the ground 40 feet from the foot of the tree. How long was the part of the tree that was broken over?

3. How many acres does a square tract of land contain, which sells for $80 an acre, and is paid for by the number of silver dollars that will lie upon its boundary?

4.* The area of a rectangular field is 30 acres, and its diagonal is 100 rods. Find its length and breadth.

5.* Suppose two candles, one of which will burn in 4 hours and the other in 5 hours, are lighted at once. How soon will one be four times the length of the other?

6.* While a log 2 feet in circumference and 10 feet long rolls 200 feet down a mountain side, a lizard on the top of the log goes from one end to the other, always remaining on top. How far did the lizard move?

7. How many calves at $3.50, sheep at $1.50, and lambs at $.50 per head, can be bought for $100, the total number bought being 100?

* Problems denoted by (*) are algebraic or geometrical. They are placed here because arithmetical solutions are often demanded.

8. A man wills to his wife $\frac{1}{3}$ of his estate, and the remaining $\frac{2}{3}$ to his son, if such should be born; but $\frac{2}{3}$ of it to the wife and the other $\frac{1}{3}$ to the daughter, if such should be born. After his death twins are born, a son and a daughter. How should the estate be divided so as to satisfy the will?

9. What is the value of $4^{3^{2^n}}$, when $n = 0$?

10. A room is 30 feet long, 12 feet wide, and 12 feet high. On the middle line of one of the smaller side walls and 1 foot from the ceiling is a spider. On the middle line of the opposite wall and 11 feet from the ceiling is a fly. The fly being paralyzed by fear remains still until the spider catches it by crawling the shortest route. How far did the spider crawl?

11. I found $10; what was my gain per cent?

12.* A conical glass is 4 inches high and 6 inches across at the top. A marble is within the glass, and water is poured in till the marble is just immersed. If the amount of water poured in is $\frac{1}{3}$ the contents of the glass, what is the diameter of the marble?

13. A banker discounts a note at 9 % per annum, thereby getting 10 % per annum interest. How long does the note run?

14. In extracting the square root of a perfect power the last complete dividend was found to be 1225. What was the power?

15.* Mr. Smith has a lawn the dimensions of which are to each other as 3 to 2. If he should increase each dimension one foot, the lawn would cover 651 square feet of land. What are the dimensions of the lawn?

16. A merchant marked his goods to gain 80 %, but on account of using an incorrect yardstick, gained only 40 %. Find the length of the measure.

17.* The area of a triangle is 24,276 square feet, and its sides are in proportion to the numbers 13, 14, and 15. Find the length of each side.

18. Between 2 and 3 o'clock, I mistook the minute hand for the hour hand, and consequently thought the time 55 minutes earlier than it was. What was the correct time?

19. A slate including the frame is 9 inches wide and 12 inches long. The area of the frame is $\frac{1}{4}$ of the whole area, or $\frac{1}{3}$ of the area inside the frame. What is the width of the frame?

20. If 6 acres of grass, together with what grows on the 6 acres during the time of grazing, keep 16 oxen 12 weeks, and 9 acres keep 26 oxen 9 weeks, how many oxen will 15 acres keep 10 weeks, the grass growing uniformly all the time?

21. A boy on a sled at the top of a hill 200 feet long, slides down and runs half as far up another hill. He sways back and forth, each time going $\frac{1}{2}$ as far as he came. How far will he have traveled by the time he comes to a halt?

22. $3 + 3 \times 3 - 3 \div 3 - 3 = ?$

23. $2 \div 2 \div 2 \div 2 \div 2 \times 2 \times 2 \times 2 \div 0 \times 2 = ?$

24. $3 \div 3 \div 3 \div 3 \times 3 \times 3 \times 0 \times 3 = ?$

25. A fly can crawl around the base of a cubical block in 4 minutes. How long will it take it to crawl from a lower corner to the opposite upper corner?

26. A squirrel goes spirally up a cylindrical post, making a circuit in each 4 feet. How many feet does it travel if the post is 16 feet high and 3 feet in circumference?

27. If the cloth for a suit of clothes for a man weighing 216 pounds costs $16, what will be the cost of enough cloth of the same quality for a man of similar form weighing 512 pounds?

28. A ball 12 feet in diameter when placed in a cubic room touches the floor, ceiling, and walls. What must be the diameter of 8 smaller balls, which will touch this ball and the faces of the given cube?

29. At what time between 3 and 4 o'clock is the minute hand the same distance from 8 as the hour hand is from 12?

30.* By cutting from a cubical block enough to make each dimension 2 inches shorter it is found that its solidity has been decreased 39,368 cubic inches. Find a side of the original cube.

31. A number increased by its cube is 592,788. Find the number.

32.* The difference of two numbers is 40; the difference of their squares is 4800. What are the numbers?

33. A man can row upstream in 3 hours and back again in 2 hours. Determine the distance, the rate of the current being 1 mile per hour.

34. A rented a farm from B, agreeing to give B $\frac{1}{5}$ of all the produce. During the year A used 90 bushels of the corn raised, and at settlement first gave B 20 bushels to balance the 90 bushels and then divided the remainder as if neither had received any. How much did B lose?

35. A certain number increased by its square is equal to 13,340. Find the number.

36.* The cube root of a certain number is 10 times the fourth root. Find the number.

37. A number divided by one more than itself gives a quotient $\frac{1}{11}$. What is the number?

38. What do I pay for goods sold at a discount of 50, 25, and 100 % off, the list price being $50?

39. If an article had cost $\frac{1}{4}$ less, the rate of loss would have been $\frac{1}{6}$ less. Find the rate of loss.

40. A merchant having been asked for his lowest prices on shoes, replied, "I give a certain per cent off for cash, the same per cent off the cash price to ministers, and the same per cent off the price to ministers to widows." The price to widows is $\frac{343}{512}$ of the marked price. What per cent does he give off for cash?

41. If James had $40 more money he could buy 20 acres of land, or with $80 less he could buy only 10 acres. How much money has he and what is the value of an acre?

42. What is the least number of gallons of wine, expressed by a whole number, that will exactly fill, without waste, bottles containing either $\frac{3}{4}$, $\frac{5}{8}$, $\frac{6}{7}$, or $\frac{5}{9}$ gallons?

43. I sold a house and gained a certain per cent on my investment. Had it cost me 20 % less, I should have gained 30 % more. What per cent did I gain?

44. Goods marked to be sold at 50 and 10 % discount were disposed of by an ignorant salesman at 60 % from the list price. What was the loss on cash sales amounting to $15,000?

45. I paid $10 cash for a bill of goods. What was the list price, if I received a discount of 50, 25, 20, and 10 % off?

46. My clock gains 10 minutes an hour. It is right at 4 P.M. What is the correct time when the clock shows midnight of the same day?

47. Two men working together can saw 5 cords of wood per day, or they can split 8 cords of wood when sawed. How many cords must they saw that they may be occupied the rest of the day in splitting it?

48. A grocery merchant sells goods at 80 % profit and takes eggs in trade at market price. If 2 eggs in each dozen are bad, find his per cent gain.

49. A hollow sphere whose diameter is 6 inches weighs $\frac{1}{8}$ as much as a solid sphere of the same material and diameter. How thick is the shell?

50. If a bin will hold 20 bushels of wheat, how many bushels of apples will it hold?

51. What per cent in advance of the cost must a merchant mark his goods so that after allowing 5 % of his sales for bad debts, and an average credit of 6 months, and 7 % of the cost of the goods for his expenses, he may make a clear gain of $12\frac{1}{2}$ % of the first cost of the goods, money being worth 6 % ?

52. A teacher in giving out the dividend 84,245,000 was misunderstood by his pupils, who reversed the order of the figures in millions period. The quotient obtained was 36,000 too small. What was the divisor?

53. Three men bought a grindstone 20 inches in diameter. How much of the diameter must each grind off so as to share the stone equally, making an allowance of 4 inches waste for the aperture?

54. James is 30 years old and John is 3 years old. In how many years will James be 5 times as old as John?

55. A merchant sold a piano at a gain of 40 %. Had it cost him $ 400 more, he would have lost 40 %. What did it cost him?

56. A steamer goes 20 miles an hour downstream, and 15 miles an hour upstream. If it is 5 hours longer in coming up than in going down, how far did it go?

57. A and B together can do a piece of work in 24 days. If A can do only $\frac{2}{3}$ as much as B, how long will it take each of them to do the work?

58. The sum of two numbers is 80; the difference of their squares is 1600. What are the numbers?

59. When a man sells goods at a price from which he received a discount of $33\frac{1}{3}$ %, what is his gain per cent?

60. $6 - 6 \div 6 + 6 \times 2 - 2 = ?$

61. $3 \div 3 \div 3 \div 3 \div 3 \div \frac{1}{3} \div \frac{1}{3} \div \frac{1}{3} \div \frac{1}{3} = ?$

62. How much water will dilute 5 gallons of alcohol 90 % strong to 30 % ?

63. I bought a house and lot for $ 1000, to be paid for in 5 equal payments, interest at 10 %, payable annually; payments to be cash, 1, 2, 3, and 4 years from date of purchase. What was the amount of each payment?

64. I buy United States 4 % bonds at 106, and sell them in 10 years at 102. What is my rate of income?

65. If a melon 20 inches in diameter is worth 20 cents, what is one 30 inches in diameter worth?

66. The difference between the true discount and the bank discount of a note due in 90 days at 6 %, is $.90. What is the face of the note?

67. A writing desk cost a merchant $ 20. At what price must it be marked so that the marked price may be reduced 40 % and still 50 % be gained?

68. A man agreed to work 12 days for $ 18 and his board, but he was to pay $ 1 a day for his board for every day he was idle. He received $ 8 for his work. How many days did he work?

69. A druggist, by selling 10 pounds of sulphur for a certain sum, gained 50 %. If the cost of sulphur advances 20 % in the wholesale market, what per cent will the druggist now gain by selling 7½ pounds for the same sum?

70.* The head of a fish is 9 inches long. The tail is as long as the head and ½ of the body, and the body is as long as the head and tail. What is the length of the fish?

71. In a corner of a bin I pour some grain which extends up the wall 8 feet, and whose base is measured by a circular line 10 feet distant from the corner. How many bushels in the pile?

72. A substance is weighed from both arms of a false balance, and its apparent weights are 4 pounds and 16 pounds. Find its true weight.

73. When wheat is worth $.90 a bushel, a baker's loaf weighs 9 ounces. How many ounces should it weigh when wheat is worth $.72 a bushel?

74. The difference between the interest of $700 and $300 for the same time at 6 % is $84. Find the time.

75. What is the price of 10 % stocks that yield a profit equal to that of 5 % bonds bought at 80?

76. If I sell oranges at 8 cents a dozen, I lose 30 cents; but if I sell them at 10 cents a dozen, I gain 12 cents. How many have I, and what did they cost me?

77. If a man can swim across a circular lake in 20 minutes, how long will it take him to ride twice around it at twice his former rate?

78. If $\frac{4}{5}$ of the time past noon, plus 4 hours, equals $\frac{3}{4}$ of the time to midnight plus 3 hours, what is the time?

79. A horse steps more than 30 and less than 50 inches at each step. If he takes an exact number of steps in walking 259 inches and an exact number in walking 407 inches, what is the length of his step?

80. I sold two horses for $200. I gained 10 % on the first and 20 % on the second. How much did each cost if the second cost $20 more than the first?

81. A thief is 27 steps ahead of an officer, and takes 8 steps while the officer takes 5; but 2 of the officer's steps are equal to 5 of the thief's. In how many steps can the officer catch him?

82. A tree is 60 feet high, which is $\frac{5}{6}$ of $\frac{6}{7}$ of the length of its shadow diminished by 20 feet. Required the length of its shadow.

83. What time is it if $\frac{1}{5}$ of the time past noon is equal to $\frac{1}{6}$ of the time to midnight?

84. Between 2 and 3 o'clock the minute and hour hands of a clock are together. What time is it?

85. Which weighs the more, a pound of feathers or a pound of gold?

86. Four pedestrians whose rates are as the numbers 2, 4, 6, and 8, start from the same point to walk in the same direction around a circular tract 100 yards in circumference. How far has each gone when they are next together?

87. If 2 miles of fence will inclose a square of 160 acres, how large a square will 3 miles of fence inclose?

88. I bought a horse for $90, sold it for $100, and soon repurchased it for $80. How much did I make by trading?

89. Considering the earth 8000, and the sun 800,000 miles in diameter, how many earths would it take to equal the sun?

90. A merchant marks his goods to sell at an advance of 25%, and sells a book for $2.25, and allows the customer 10% off from the marked price. What did the book cost the merchant?

91. A merchant gives a discount of 10%, but uses a yard measure .72 of an inch too short. What rate of discount would allow him the same amount of gain if the measure were correct?

92.* A merchant at one straight cut took off a segment of a cheese which weighed 2 pounds, and had $\frac{1}{6}$ of the circumference. What was the weight of the whole cheese?

93. What is the shortest distance that a fly will have to go, crawling from one of the lower corners of the room to the opposite upper corner — the room being 20 feet long, 15 feet wide, and 10 high?

94. I buy goods at 50% off and sell them at 40 and 10% off. What is my per cent profit?

95. A farmer goes to a store and says: "Give me as much money as I have and I will spend ten dollars with you." It is given him, and the farmer repeats the operation to a second, and a third store, and has no money left. What did he have in the beginning?

96. A book and a pen cost \$1.20; the book cost \$1 more than the pen. What was the cost of each?

97. A dealer asked 30% profit, but sold for 10% less than he asked. What per cent did he gain?

98. Suppose we leave the Pacific coast at sunrise, on September 28, and cross the Pacific Ocean fast enough to have sunrise all the way over to Manila, where it is sunrise September 29. How do you account for the lost day?

99. A man was asked whether he had a score of sheep. He replied, "No, but if I had as many more, half as many more, and two sheep and a half, I should have a score." How many had he?

100. What part of threepence is a third of twopence?

101. Three boys met a servant maid carrying apples to market. The first took half of what she had, but returned to her 10; the second took $\frac{1}{3}$, but returned 2; and the third took away half those she had left, but returned 1. She then had 12 apples. How many had she at first?

102. A person having about him a certain number of German coins, said, "If the third, fourth, and sixth of them were added together, they would make 54." How many did he have?

103. If a log starts from the source of a river on Friday, and floats 80 miles down the stream during the day, but comes back 40 miles during the night with the return tide, on what day of the week will it reach the mouth of the river, which is 300 miles long?

104. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 0 = ?$

105. One gentleman meeting another and inquiring the time past 12 o'clock, received for an answer, "One third of the time from now to midnight." What time in the afternoon was it?

106. A said to B, "Give me $100, and then I shall have as much as you." B said to A, "Give me $100, and then I shall have twice as much as you." How many dollars had each?

107. At the rate of 4 miles per hour, a raft floats past the landing at 8 A.M.; the down-going steamer, at the rate of 16 miles per hour, passes the landing at 4 P.M. What time is it when the steamer overtakes the raft?

108. A bought a horse for $80 and sold it to B at a certain rate per cent of gain. B sold it to C at the same rate per cent of gain. C paid $105.80 for the horse. What price did B pay, and what was the rate per cent of gain?

109. The sum of two numbers is 582 and their difference is 218. What are the numbers?

110. What are the contents and inside surface of a cubical box whose longest inside measurement is 2 feet?

111. Three persons engaged in a trade with a joint capital of $9000. A's capital was in trade 5 months, B's 2 months, and C's 1 month A's share of the gain was $450, B's $270, and C's $180. What was the capital of each?

112. A man was hired for a year for $100 and a suit of clothes, but at the end of 8 months he left and received his clothes and $60 in money. What was the value of the suit of clothes?

113. A note for $100 was due on September 1, but on August 11, the maker proposed to pay as much in advance as would allow him 60 days after September 1, to pay the balance. How much did he pay August 11, money being worth 6%?

114. If I rent a house at $18 a month, payable monthly in advance, what amount of cash payable at the beginning of the year will pay the year's rent, interest at 5%?

115. If a house rents for $20 a month, payable at the close of each month, what amount is due if not paid till the end of year, interest at 6%?

116. A merchant sold a lease of $480 a year, payable quarterly, having 8 years and 9 months to run, for $2500. Did he gain or lose, and how much, interest at 8%, payable semiannually?

117. A box of oranges weighed 64 pounds by the grocer's scales, but being placed in the other scale of the balance, it weighed only 30 pounds. What was the true weight of the box of oranges?

118. If a ball 5 inches in diameter weighs 8 pounds, what will be the weight of a similar ball 10 inches in diameter?

119. A, B, and C dine on 8 loaves of bread. A furnishes 5 loaves, B 3 loaves, and C pays the others 8 cents for his share. How must A and B divide the money?

120. A boy being asked how many fish he had, replied, "11 fish are 7 more than $\frac{2}{5}$ of the number." How many had he?

121. I have two lamps, one of 4-candle power, and one of 9-candle power. If the former is 30 feet distant, how far away must I place the latter to give me the same amount of light?

122. A merchant bought 90 boxes of lemons for $265, paying $3.50 for first quality and $3 for second quality. How many boxes of each kind did he buy?

123. A vessel after sailing due north and due east on alternate days, is found to be $16\sqrt{2}$ miles northeast of the starting place. What distance has it sailed?

124. Two teachers work together; for 10 days' work of the first and 8 days' work of the second they receive $28, and for 5 days' work of the first and 11 days' work of the second they receive $21. What is each man's daily wages?

125. A hind wheel of a carriage 4 feet 6 inches high revolved 720 times in going a certain distance. How many revolutions did the fore wheel make, which was 4 feet high?

126. A farmer carried some eggs to market, for which he received $2.56, receiving as many cents a dozen as there were dozen. How many dozen were there?

127. Three men, A, B, and C, are to mow a circular meadow containing 9 acres. A is to receive $3, B $4, and C $5 for his work. What width must each man mow?

128. If the diameter of a cannon ball is 100 times that of a bullet, how many bullets will it take to equal the cannon ball?

129. A man sells a cow and a horse for $120. He sells the horse for $100 more than the cow. What did he sell each for?

130. If a man 5½ feet tall weighs 166.375 pounds, how much will a man 6 feet tall of similar proportions weigh?

131. Having sold a house and lot at 4% commission, I invest the net proceeds in merchandise after deducting my commission of 2% for buying. My whole commission is $50. For how much did I sell the house and lot?

132. A teacher agreed to teach a 10-weeks school for $100 and his board. At the end of the term, on account of 3 weeks' absence caused by sickness, he received only $58. What was his board per week?

133. In buying a bill of goods, I am offered my choice of 50, 25, and 5% discount, or 5, 25, and 50% discount. Which is better?

134. The product of two numbers exceeds their difference by their sum. Find one of the numbers.

135. Twice the sum of two numbers plus twice their difference is 80. What is the greater number?

136. One half the sum of two numbers exceeds one half their difference by 60. What is the smaller number?

137. What per cent is gained by selling 13 ounces of coffee for a pound?

138. If I sell $\frac{5}{6}$ of an acre of land for what an acre cost me, what per cent do I gain?

139. I sold a horse for $200, losing 20 %; I bought another and sold it at a gain of 25 %; I neither gained nor lost on the two. What was the cost of each?

140. At the time of marriage a wife's age was $\frac{5}{6}$ of the age of her husband, and 24 years after marriage her age was $\frac{11}{12}$ of the age of her husband. How old was each at the time of marriage?

141. How much water is there in a mixture of 50 gallons of wine and water, worth $ 2 per gallon, if 50 gallons of the wine costs $250?

142. A Texas farmer keeps 2100 cows on his farm. For every 3 cows he plows 1 acre of ground and for every 7 cows he pastures 2 acres of land. How many acres are in his farm?

143. The divisor is 6 times the quotient. Find the quotient.

144. When gold was worth 25 % more than paper money, what was the value in gold of a dollar bill?

145. I bought 15 yards of ribbon, and sold 10 of them for what I paid for all, and the remainder at cost. I gained $.25 by the transaction. What did the ribbon cost me?

146. If a ball of yarn 6 inches in diameter makes one pair of gloves, how many similar pairs will a ball 12 inches in diameter make?

147. At what time between 4 and 5 o'clock do the hour and minute hands of a clock coincide?

148. At what time between 2 and 3 o'clock do the hour and minute hands of a clock coincide?

149. At what time between 2 and 3 o'clock are the hour and minute hands of the clock at right angles?

150. At what time between 2 and 3 o'clock are the hands of a clock exactly opposite each other?

151. From 200 hundredths take 15 tenths.

152. Find the sum of 2324 thousandths and 24,325 hundredths.

153. A lady at her marriage had her husband agree that if at his death they should have only a daughter, she should have $\frac{4}{5}$ of his estate; and if they should have only a son, she should have $\frac{2}{5}$. They had a son and a daughter. How much should each receive, if the estate was worth $23,375?

154. A crew can row 24 miles downstream in 3 hours, but requires 4 hours to row back. What is the rate of the current?

155. What minuend is 80 greater than the subtrahend, which is 20 greater than the remainder?

156. The G. C. D. of two numbers is 60 and the L. C. M. is 720. Find the product of the numbers.

157. In extracting the cube root of a perfect power the operator found the last complete dividend to be 132,867. Find the power.

158. A merchant marks his goods at an advance of 25 % on cost. After selling $\frac{1}{3}$ of the goods, he finds that some of the goods on hand are damaged so as to be worthless; he marks the salable goods at an advance of 10 % on the marked price and finds in the end that he has made 20 % on cost. What part of the goods was damaged?

159. A king has a horse shod and agrees to pay 1 cent for driving the first nail, 2 cents for the second, 4 cents for the third, doubling each time. What will the shoeing with 32 nails cost?

160. I sold a book at a loss of 25 %. Had it cost me $1 more, my loss would have been 40 %. Find its cost.

161. At noon the three hands — hour, minute, and second — of a clock are together. At what time will they first be together again?

162. A train is traveling from one station to another. After traveling an hour it breaks down and is delayed for an hour. It then proceeds at $\frac{3}{5}$ of its former speed, and arrives 3 hours late. Had it gone 50 miles farther before the breakdown, it would have arrived 1 hour and 20 minutes sooner. Find the rate of the train and the distance between the stations.

163. If a cocoanut 4 inches in diameter is worth 5 cents, what is the worth of one 6 inches in diameter?

164. Prove that the product of the G. C. D. and L. C. M. of two numbers is equal to the product of the numbers.

165. Sum to infinity the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$.

166. Find the sum of $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$ to infinity.

167. Find the sum of $4 + 0.4 + 0.04 + \cdots$ to infinity.

168. What is the distance passed through by a ball before it comes to rest, if it falls from a height of 100 feet and rebounds half the distance at each fall?

169. Two trains start at the same time, one from Jacksonville to Savannah, the other from Savannah to Jacksonville. If they arrive at destinations 1 hour and 4 hours after passing, what are their relative rates of running?

170. If sound travels at the rate of 1090 feet per second, how far distant is a thundercloud when the sound of the thunder follows the flash of lightning after 10 seconds?

171. The G. C. D. and the L. C. M. of two numbers between 100 and 200 are respectively 4 and 4620. Find the numbers.

172. What three equal successive discounts are equivalent to a single discount of 58.8 %?

173. How much will the product of two numbers be increased by increasing each of the numbers by 1?

174. I can beat James 4 yards in a race of 100 yards, and James can beat John 10 yards in a race of 200 yards. How many yards can I beat John in a race of 500 yards?

175. Three ladies own a ball of yarn 6 inches in diameter. What portion of the diameter must each wind off in order to divide the yarn equally among them?

176. Demonstrate the following: If the greater of two numbers is divided by the less, and the less is divided by the remainder, and this process is continued till there is no remainder, the last divisor will be the greatest common divisor.

177. Find the volume of a rectangular piece of ice 8 feet long, 7 feet wide, and floating in water, with 2.4 inches of its thickness above water, the specific gravity of ice being .9.

178. Two trains, 400 and 200 feet long respectively, are moving with uniform velocities on parallel rails; when they move in opposite directions they pass each other in 5 seconds, but when they move in the same direction, the faster train passes the other in 15 seconds. Find the rate per hour at which each train moves.

179. A boy is running on a horizontal plane directly towards the foot of a tree 50 feet in height. When he is 100 feet from the foot of the tree, how much faster is he approaching it than the top?

180. Express 77,610 in the duodecimal scale.

181.* In what scale is 6 times 7 expressed by 110?

182. Express Adam's age at his death in the binary scale.

183. Add 3152_6, 4204_6, 3241_6, 3103_6.

184. Subtract $12{,}312_5$ from $23{,}024_5$.

185. Multiply $62{,}453_7$ by 325_7.

186. Divide $2{,}034{,}431_5$ by 234_5.

187. Extract the square root of 170_9.

188.* Extract the cube root of 3120_4.

189. How many trees can be set out upon a space 100 feet square, allowing no two to be nearer each other than 10 feet?

190. How many stakes can be driven down upon a space 12 feet square, allowing no two to be nearer each other than 1 foot?

191. Multiply 789,627 by 834, beginning at the left to multiply.

192. Two fifths of a mixture of wine and water is wine; but if 10 gallons of water be added to it, then only $\frac{7}{20}$ of the mixture will be wine. How many gallons of each liquid is in the mixture?

193. Simplify

$$10 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 - \frac{1}{3}}}}.$$

194. 15,600 is the product of three consecutive numbers. What are they?

195. Find a number which is as much greater than 1042 as it is less than 1236.

196. Multiply 729,038 by 105,357 using only 3 multipliers.

197. What is the smallest number to be subtracted from 10,697 to make the result a perfect cube?

198. I wish to reach a certain place at a certain time; if I walk at the rate of 4 miles an hour, I shall be 10 minutes late, but if I walk 5 miles an hour, I shall be 20 minutes too soon. How far have I to walk?

199. A wineglass is half full of wine, and another twice the size is $\frac{1}{3}$ full. They are then filled up with water, and the contents mixed. What part of the mixture is wine, and what part water?

200. A cork globe 2 feet in diameter, whose specific gravity is $\frac{1}{36}$, is hollowed out and filled with lead whose specific gravity is 10. What must be the thickness of the shell of cork so that it will sink just even with the surface of the water?

201. What temperature will result from mixing 100 pounds of ice at 14° F. with 80 pounds of steam at 270° F.?

202. It is 1800 miles from *A* to *C*, and the "Sunset Flyer" annihilates the distance in 50 hours. She averages 30 miles an hour from *A* to *B*, and 55 miles an hour from *B* to *C*. Locate *B*.

203. A square and its circumscribing circle revolve about the diagonal of the square as an axis. Compare the volumes and surfaces of the solids generated, the diagonal being 6 feet.

204. The aggregate area of two square fields is $8\frac{1}{8}$ acres. The side of the second is 10 rods longer than that of the first. Ascertain the length of the first.

205. How high above the earth's surface (radius 4000 miles) would a pound weight weigh but one ounce avoirdupois by a scale indicator, corrected for change of elasticity by temperature?

206. On a west-bound freight train a man is running eastward at the rate of 6 miles an hour, and likewise a man runs in the same direction 8 miles an hour on a train going east. If the trains pass while running 36 and 22 miles an hour, respectively, how many miles apart are the men at the end of one minute from the moment they pass each other?

207. A drawer made of inch boards is 8 inches wide, 6 inches deep, and slides horizontally. How far must it be drawn out to put into it a book 4 inches wide and 9 inches long?

208. The dividend is 4352, the remainder 17, which is the G.C.D. of the quotient and divisor, whose difference you may find.

209. B paid $9 more than true discount by borrowing money at a bank for one year at 12%. Find the face of the note.

210. How many feet of inch lumber in a wagon tongue 10 feet long, 4 inches square at one end and 2 inches by 3 inches at the other end?

211. How many inch balls can be put in a box which measures inside 10 inches square and 5 inches deep?

212. If the posts of a wire fence around a rectangular field twice as long as wide were set 16 feet apart instead of 12 feet, it would save 66 posts. How many acres in the field?

213. If gold is 19.3 times as heavy as water and copper 8.89 as heavy, how many times as heavy is a coin composed of 11 parts of gold and 1 part of copper?

214. A ball falls 15 feet and bounces back 5 feet. How far will it bound before it comes to rest?

215. A borrows $500 from a building and loan association and agrees to pay $9.50 per month for 72 months, the first payment to be made at the end of the first month. What rate of interest does he pay? The association claims to charge only 8% (the legal rate in Alabama). How can the per cent be figured out?

216. A rope 50 feet long is fastened to two stakes, driven 40 feet apart. A calf is fastened to a ring which moves freely on this rope. Over what area can the calf graze?

217. A metal dog made of gold and silver weighs 8.75 ounces. Its specific gravity is 14.625, that of gold 19.25, and that of silver 10.5. Find the number of ounces of gold in it.

218. By drilling an inch hole through a cubical block of wood parallel to the faces of the block, $\frac{1}{60}$ of the wood was cut away. What were the dimensions of the block?

219. Find two numbers whose G.C.D. is 24, and L.C.M. 288.

220. Find the greatest number that will divide 364, 414, and 539, and leave the same remainder in each case.

221. Had an article cost me 8% less, the number of per cent gain would have been 10% more. What was the gain?

222. At what time between 3 and 4 o'clock will the minute hand be as far from 12 on the left side of the dial plate as the hour hand is from 12 on the right side?

223. A ball whose specific gravity is $3\frac{3}{8}$ measures a foot in diameter. Find the diameter of another ball of the same weight but with a specific gravity of $2\frac{10}{27}$.

224. A owes $2500 due in two years. He pays $500 cash and gives a note payable in 8 months, for the balance. Find the face of the note, money being worth 6%.

225. A man bought a horse for $201, giving his note due in 30 days. He at once sold the horse, taking a note for $224.40, due in 4 months. What was his rate of gain at the time of the sale, interest 6%?

226. The minute hand and the hour hand coincide every 65 minutes. Does the clock gain or lose, and how much?

227. A ball weighing 970 ounces, weighs in water 892 ounces, and in alcohol 910 ounces. What is the specific gravity of alcohol?

228. A steamer moves through 8° of longitude daily in plying to and fro across the Atlantic. How long is it from one noon to the next?

229. A, B and C raise 165 acres of grain. A owns 100 acres of the land and B 65 acres. C pays the others $110 rent. How must A and B divide this money if the grain is shared equally?

230. A silver cup is a hemisphere filled with wine worth $1.20 a quart. The value of the cup is 2 dimes for every square inch of internal surface, and the cup is worth just as much as the wine. What is the value of the cup?

231. A ball 12 inches in diameter is rolled around a circular room 12 feet in diameter in such a way that it always touches both wall and floor. How many revolutions does the ball make in rolling once around the room?

232. A man desires to purchase eggs at 5 cents, 1 cent, and $\frac{1}{2}$ cent, respectively, in such numbers that he will obtain 100 eggs for a dollar. How many solutions in rational integers?

233. How many board feet in a piece of lumber, 2 inches square at one end and at the other end 1 inch by 12 inches, if the ends are parallel?

234. How many board feet in the above piece of lumber if it is 24 feet long?

235. Is anything expressed by $.\frac{1}{2}$? If so, what?

236. A man bequeathed to his son all the land he could inclose in the form of a right-angled triangle with 2 miles of fence, the base of the triangle to be 128 rods. How many acres did he get?

237. The distance around a rectangular field is 140 rods, and the diagonal is 50 rods. Find its length, breadth, and area.

238. The specific gravity of ice being .918 and of sea water 1.03, find the volume of an iceberg floating with 700 cubic yards above water.

239. A room is 30 feet long, 12 feet wide, and 12 feet high. At one end of the room, 3 feet from the floor, and midway from the sides, is a spider. At the other end, 9 feet from the floor, and midway from the sides, is a fly. Determine the shortest path by way of the floor, ends, sides, and ceiling, the spider can take to capture the fly.

240. A and B are engaged in buying hogs, each paying out of his individual funds for hogs purchased by him, and each retaining as his individual funds the money received from sales made by him. They now wish to form a partnership to cover all past transactions and to share equally in the settlement for sales and purchases, and also to be equally interested in hogs which they have on hand unsold. The following data given:

A has paid for hogs $1183.35, and received from sales of hogs $434.35.

B has paid for hogs $241.55, and received from sales of hogs $619.00.

Invoice of hogs on hand at this time $511.35.

How much does A owe B, or B owe A, so that they will have shared equally in payments and receipts, and be equally interested in the hogs on hand?

241. The hour, minute, and second hands of a clock turn on the same center. At what time after 12 o'clock is the hour hand midway between the other two? The second hand midway between the other two? The minute hand midway between the other two?

242. My agent sold pork at a commission of 7%. The proceeds being increased by $6.20, I ordered him to buy cattle at a commission of $3\frac{1}{3}$%. Cattle now declined in price $33\frac{1}{3}$%, and I found my total loss, including commissions, to be exactly $1002.20. Find the value of the pork.

243. A owes $900, due December 10, but he makes two equitable payments, one September 8 and the other January 10. Find each payment.

244. A man, dying, left an estate of $23,480 to his three sons, aged 15, 13, and 11 years, to be so divided that each share placed at interest shall amount to the same sum as the sons, respectively, become 21 years of age. What was each son's share, money being worth 5%?

245. A man spent $100 in buying two kinds of silk at $4.50 and $4.00 a yard; by selling it at $4.25 per yard he gained 2%. How many yards of each did he buy?

246. A lady being asked the time of day replied, "It is between 4 and 5 o'clock, and the hour and minute hands are together." What was the time?

247. Three men A, B, and C can do a piece of work in 60 days. After working together 10 days, A withdraws and B and C work together at the same rate for 20 days, then B withdraws and C completes the work in 96 days, working $\frac{1}{3}$ longer each day. Working at his former rate, C alone could do the work in 222 days. Find how long it would take A and B each separately to do the work.

248. In a class there are twice as many girls as boys. Each girl makes a bow to every other girl, to every boy, and to the teacher. Each boy makes a bow to every other boy, to every girl, and to the teacher. In all there are 900 bows made. How many boys are in the class? [illegible]

249. A boy weighing 96 pounds is seated on one end of a seesaw 16 feet long, and a boy weighing 120 pounds is seated on the other end. Find the distance of each boy from the point of support, the lengths of the two arms of the plank being inversely proportional to the weights at their ends.

250. Two men are on opposite sides of the center of the earth. Find the shortest distance that each will be required to go in order to exchange places, provided they travel different routes and so travel as to enjoy each other's company for 500 miles of the distance. (Radius of earth = 4000 miles.)

251. A conical wine glass 2 inches in diameter and 3 inches deep is $\frac{8}{27}$ full of water. What is the depth of the water?

252. A hollow sphere 8 inches in diameter is filled with water. How many hollow cones, each 8 inches in altitude, and 8 inches in diameter at the base, can be filled with the water in the sphere?

ALGEBRAIC PROBLEMS

1. I am now twice as old as you were when I was your age. When you are as old as I now am, the sum of our ages will be 100. What are our ages?

2. A starts from Gunter to Denton, and at the same time B starts from Denton to Gunter; A reaches Denton 32 hours, and B reaches Gunter 50 hours, after they meet on the way. In how many hours do they make the journey?

3. At what time between 10 and 11 o'clock is the second hand of a clock one minute space nearer to the hour hand than it is to the minute hand?

4. In walking along a street on which electric cars are running at equal intervals from both ends, I observe that I am overtaken by a car every 12 minutes, and that I meet one every 4 minutes. What are the relative rates of myself and the cars, and at what intervals of time do the cars start?

5. What are eggs per dozen when 2 less in a shilling's worth raise the price one penny per dozen?

6. Two men agree to build a walk 100 yards in length for $200. They divide the work so that one man should receive 50 cents more per yard than the other. How many yards does each man build, if he receives $100?

7. Two boats start from opposite sides of a river at the same instant, and throughout the journeys to be described maintain their respective speed. They pass one another at a point just 720 yards from the left shore. Continuing on their respective journeys, they reach opposite banks, where each boat remains 10 minutes and then proceeds on its return trip.

This time the boats meet at a point 400 yards from the right shore. What is the width of the river?

8. How many acres does a square tract of land contain, which sells for $160 an acre, and is paid for by the number of silver dollars that will lie upon its boundary?

9. Two girls, 4 feet apart, walk side by side around a circular park. How far does each walk if the sum of their distances is 1 mile?

10. How many acres are there in a field, the number of rails used in fencing the field equaling the number of acres — each rail being 11 feet long and the fence 4 rails high?

11. Three men are going to make a journey of 40 miles. The first can walk at the rate of 1 mile per hour, the second walks at the rate of 2 miles per hour, and the third goes in a buggy at the rate of 8 miles per hour. The third takes the first with him and carries him to such a point as will allow the third time to drive back to meet the second, and carry him the remaining part of the 40 miles, so as all may arrive at the same time. How long will it require to make the journey?

12. Two trains, 400 and 200 feet long respectively, are moving with uniform velocities on parallel rails; when they move in opposite directions, they pass each other in 5 seconds, but when they move in the same direction, the faster train passes the other in 15 seconds. Find the rate per hour at which each train moves.

13. How many minutes is it until 6 o'clock, if 50 minutes ago it was 4 times as many minutes past 3 o'clock?

14. A man bought a gun for a certain price. Now, if he sells it for $9, he will lose as much per cent as the gun cost. Required the cost of the gun.

15. In a nest were a certain number of eggs; if I had brought 1 egg that I didn't bring, I should have brought $\frac{2}{3}$ of them, and if I had left 2 eggs that I did bring, I should have brought half of them. How many eggs were in the nest?

16. A man sold a lot for $ 144. The number of dollars the lot cost was the same as the number of per cent profit. What did the lot cost?

17. What is the side of a cube which contains as many cubic inches as there are square inches in its surface?

18. What is the length of one edge of that cube which contains as many solid units as there are linear units in the diagonal through the opposite corners?

19. The sum, the product, and the difference of the squares of two numbers are all equal. Find the numbers.

20. Upon inquiring the time of day, a gentleman was informed that the hour and minute hands were together between 4 and 5. What was the time of day?

21. An officer wishing to arrange his men in a solid square, found by his first arrangement that he had 39 men over. He then increased the number of men on a side by 1, and found 50 men were needed to complete the square. How many men did he have?

22. A young lady being asked what she paid for her eggs, replied, "Three dozen cost as many cents as I can buy eggs for 36 cents." What was the price per dozen?

23. A cube is formed out of a lot of cubical blocks, 1 foot each, and it is found by using 448 more another cube is formed, the edge of which is 8 feet. What was the length of an edge of the original cube?

24. Find two numbers whose product is equal to the difference of their squares, and the sum of their squares equal to the difference of their cubes.

25. A young lady being asked her age, answered, "If you add the square root of my age to $\frac{3}{8}$ of my age, the sum will be 10." Required her age.

26. There is a fish whose head is 9 inches long; the tail is as long as the head and $\frac{1}{2}$ the body; and the body is as long as the head and the tail together. What is the length of the fish?

27. I bought 2 horses for $80; I sold them for $80 apiece, the gain on the one being 20 % more than on the other. What was the cost of each?

28. A man has a square lot upon which he wishes to build a house facing the street, with a driveway around the other three sides. He wants the house to cover the same amount of land as the driveway. How wide shall he make the driveway, the lot being 100 feet each way?

29. An officer can form his men into a hollow square 4 deep, and also into a hollow square 6 deep; the front in the latter formation contains 12 men fewer than in the former formation. Find the number of men.

30. How must a line 12 inches long be divided into two parts so that the rectangle of the whole line and one part shall equal the square on the other side?

31. Two miners, B and C, have the same monthly wages. B is employed 7 months in the year, and his annual expenses are $350; C is employed 5 months in the year, and his annual expenses are $250. In 5 years B saves the same amount that C saves in 7 years. What were the monthly wages of each?

32. Simplify: $\frac{3^{n^2+1}}{3^{(n+2)^2}} \div \frac{2^{(n)^0}}{9^{(n-1)}}$.

33. Find the value of x in the equation:

$$2(1+x^4)=(1+x)^4.$$

34. Solve the equation:

$$x^4+4\,m^3x-m^4=0.$$

Solve the following equations:

35. $x^2 + y = 11,$
$y^2 + x = 7.$

36. $x^2 - y = y^2 + x,$
$x^2 + y = 5(x - y^2).$

37. $x + y = 10,$
$y\sqrt{x} = 12.$

38. $x^2 + y^2 = 13,$
$y + xy = 9.$

39. $\sqrt{x} + \sqrt{y} = 5,$
$x + \sqrt{xy} = 10.$

40. $x + y = 13,$
$\sqrt{x} + \sqrt{xy} = 8.$

41. $x^2 + xy + y^2 = 39,$
$x^2 + xz + z^2 = 19,$
$y^2 + yz + z^2 = 49.$

42. $5\,y(x^6 + 1) - 3\,x^3(y^2 + 1) = 0,$
$15\,y^3(x^2 + 1) - x(y^6 + 1) = 0.$

43. A farmer being asked how many acres he had, replied, "My land is square. I have plowed just 2 rods wide around, and have plowed just $\frac{1}{2}$ my land." How many acres has he?

44. From a 10-gallon keg of wine a man filled a jug. He then filled the keg with water, and repeated the operation a second time, when he found the keg contained equal amounts of water and wine. Find the capacity of the jug.

45. If a certain number is divided by 32, the remainder is 25; if divided by 25, the remainder is 19; and if divided by 19, the remainder is 11. What is the number?

46. If Dr. A loses 3 patients out of 7; Dr. B, 4 out of 13; and Dr. C, 5 out of 19; what chance has a sick man for his life, who is dosed by the three doctors for the same disease?

47. Said Robin to Richard, "If ever I come
To the age that you are, brother mine,
Our ages united would amount to the sum
Of years making ninety-nine."
Said Richard to Robin, "That's certain, and if it be fair
For us to look forward so far,
I then shall be double the age that you were,
When I was the age that you are."

48. A tells the truth 2 times out of 3, B 6 times out of 7, and C 4 times out of 5. What is the probability of the truth of an assertion that A and B affirm and C denies?

49. A plank 16 feet long with a weight of 196 pounds, on one end balances across a fulcrum placed 1 foot from the 196-pound weight. What is the weight of the plank?

50. A man desires to purchase eggs at 5 cents, 1 cent, and $\frac{1}{2}$ cent, respectively, in such numbers that he will obtain 100 eggs for a dollar. How many solutions in rational integers?

51. Ann's brother started to school. On the first day the teacher asked him his age. He replied, "When I was born, Ann was $\frac{1}{4}$ the age of mother and is now $\frac{1}{3}$ as old as father, and I am $\frac{1}{4}$ of mother's age. In 4 years I shall be $\frac{1}{4}$ as old as father." How old is Ann's brother?

52. Solve for x:

$$x^6 = \frac{2^{m+6} \cdot 8^{m-1} \cdot 9^{(n+1)^2}}{3^{-n^2+7n-4} \cdot 4^{(1+m)^2} \cdot 16^0} \div (27^{n^2-n} \cdot 32^{-\frac{2}{5}m^2-1}).$$

53. My wife was born

$$\text{June } \left\{\left[\frac{16}{(\frac{3}{2})^{\frac{1}{3}}}\right]\left[\frac{2^{a(a-1)}}{2^{a+1}} \cdot \frac{4^{a+1}}{2^{a^2-1}}\right]\left[\frac{(3^2 \cdot 3 \cdot 2^2)^{\frac{1}{3}}(3)^{\frac{1}{2}}}{(2 \cdot 3 \cdot 3^{\frac{1}{2}} \cdot 6^2)^{\frac{1}{3}}}\right]\right\}^{1/2}, 1887.$$

What was her age August 10, 1904?

NOTE. — Problems 54–67, inclusive, are from Bowser's "College Algebra."

54. Express with positive exponents

$$\sqrt[3]{(a+b)^5} \times (a+b)^{-\frac{2}{3}}.$$

55. Extract the square root of

$$6 + 2\sqrt{2} + 2\sqrt{3} + 2\sqrt{6}.$$

56. Extract the square root of

$$5 + \sqrt{10} - \sqrt{6} - \sqrt{15}.$$

57. Solve $x^{-\frac{1}{2}} + x^{-\frac{1}{4}} = 6.$

58. Solve $x^{\frac{5}{6}} + x^{\frac{5}{3}} = 1056$.

59. Solve $\dfrac{x^2}{a^{\frac{1}{2}} + b^{\frac{1}{2}}} - (a^{\frac{1}{2}} - b^{\frac{1}{2}})x = \dfrac{1}{(ab^2)^{-\frac{1}{2}} + (a^2b)^{-\frac{1}{2}}}$.

60. Solve the following:

$$6(x^2 + y^2 + z^2) = 13(x + y + z) = \tfrac{481}{6},$$
$$xy = z^2.$$

61. $x^4 + y^4 = 14\,x^2y^2,$
$x + y = a.$

62. $\left(\dfrac{3\,x}{x+y}\right)^{\frac{1}{2}} + \left(\dfrac{x+y}{3\,x}\right)^{\frac{1}{2}} = 2,$
$xy - x - y = 54.$

63. $x^3 + y(xy - 1) = 0,$
$y^3 - x(xy + 1) = 0.$

64. $x^5 + y^5 = xy(x + y)^3,$
$xy^4 = (x + y)^3.$

65. $(x^6 + 1)y = (y^2 + 1)x^3,$
$(y^6 + 1)x = 9(x^2 + 1)y^3.$

66. A offers to run three times round a course while B runs twice round, but A gets only 150 yards of his third round finished when B wins. A then offers to run four times round to B three times, and now quickens his pace so that he runs 4 yards in the time he formerly ran 3 yards. B also quickens his so that he runs 9 yards in the time he formerly ran 8 yards, but in the second round falls off to his original pace in the first race, and in the third round goes only 9 yards for 10 he went in the first race, and accordingly this time A wins by 180 yards. Determine the length of the course.

67. On the ground are placed n stones; the distance between the first and second is 1 yard, between the second and third 3 yards, between the third and fourth 5 yards, and so on. How far will a person have to travel who shall bring them one by one to a basket placed at the first stone?

68. Sionius and his wife Lionius sip from the same bowl filled with milk. Lionius sips during $\frac{2}{5}$ of the time which Sionius would take to empty the bowl; then Lionius stops and

hands it to Sionius to finish. If both had sipped together, the bowl would have been emptied 6 minutes sooner, and Lionius would have received $\frac{2}{3}$ of the milk which Sionius sipped after receiving the bowl from Lionius. In what time would Sionius and Lionius sipping together empty the bowl?

69. Once, in classic days, Silenus lay asleep, a goatskin filled with wine near him. Dionysius passing by, profited by seizing the skin, and drinking for $\frac{2}{3}$ of that time in which Silenus alone could have emptied said skin. At this point Silenus awoke, and seeing what was happening, snatched away the precious skin, and finished it.

Now, had both started together, and drunk simultaneously, they would have consumed the wine skin in 2 hours less time. And, in this case, Dionysius' share would have been $\frac{1}{2}$ as much as Silenus did secure, by waking and snatching the skin. In what time would either one of them alone finish the goatskin?

70. Three regiments move north as follows: B is 20 miles east of A; C is 20 miles south of B, and each marches 20 miles between the hours of 5 A.M. and 3 P.M. A horseman with a message from C starts at 5 A.M. and rides north till he overtakes B, then sets a straight course for the point at which he calculates to overtake A, then sets a straight course for the next point at which he will again overtake B, then rides south to the point where he first overtook B, reaching that point at the same time as C, namely, 3 P.M. What uniform rate of travel enabled the messenger to do this?

71. Three men and a boy agree to gather the apples in an orchard for $50. The boy can shake the apples in the same time that the men can pick them, but any one of the men can shake them 25 % faster than the other two men and boy can pick them. Find the amount due each.

GEOMETRICAL EXERCISES

1. Construct a trapezoid having given the sum of the parallel sides, the sum of the diagonals, and the angle formed by the diagonals.

2. If three equal circles are tangent to each other, each to each, and inclose a space between the three arcs equal to 200 square feet, find the diameter of each circle.

3. An iron rod of a certain length stands against the side of a house; if it is pulled out 4 feet at the bottom, the top moves down the side of the house a distance equal to $\frac{1}{5}$ the rod. Find the length of the rod.

4. A circle whose area is 1809.561 square feet is described upon the perpendicular of a right triangle as a diameter. From the point where the circumference cuts the hypotenuse a tangent to the circle is drawn, which cuts the base. If the shortest distance from the point of intersection of the tangent with the base to the perpendicular is 18 feet, what is the length of the hypotenuse?

5. The number of cubic inches contained by two equal opposite spherical segments, together with the number of cubic inches contained by the cylinder included between these segments, is 600. If this be $\frac{2}{3}$ of the number of cubic inches contained by the whole sphere, find the height of the cylinder.

6. The sum of the sides of a right-angled triangle is 200 feet. What is its area, the hypotenuse being 4 times the perpendicular let fall upon it from the right angle?

7. In a right-angled triangle the hypotenuse is 100 feet, and a line bisecting the right angle and terminating in the hypotenuse is 14.142 feet. Find the length of each of the other two sides.

8. Two posts, one of which is 24, and the other 16 feet high are 100 feet apart. What is the length of a rope just long enough to touch the ground between them, the ends of the rope being fastened to the top of each post?

9. A ladder 30 feet long leans against a perpendicular wall at an angle of 30°. How far will its middle point move, provided the top moves down the wall until it reaches the ground?

10. A man owns a piece of land in the form of a right-angled triangle. The sum of the sides about the right angle is 70 feet and their difference equals the length of a line parallel to the shorter side, dividing the triangle into two equal parts. Determine the length of the shorter side.

11. Required the greatest right triangle which can be constructed upon a given line as hypotenuse.

12. A man has a lot the shape of which is an equilateral triangle, with an area of 60 square rods. How long a rope will be required to graze his horse over $\frac{1}{3}$ the lot, provided he ties the rope to a corner post?

13. An iron ball 3 inches in diameter weighs 8 pounds. Find the weight of an iron shell 3 inches thick, whose external diameter is 30 inches.

14. Find the altitude of the maximum cylinder that can be inscribed in a cone whose altitude is 9 feet and whose base is 6 feet.

15. Construct a plane triangle having given the base, the vertical angle, and the bisector of the vertical angle.

16. How much of the earth's surface would a man see if he were raised to the height of the diameter above it?

17. To what height must a man be raised above the earth in order that he may see $\frac{1}{4}$ of its surface?

18. What part of the surface of a sphere 20 feet in diameter is illuminated by a lamp 100 feet from the surface of the sphere?

19. If the earth is assumed to be a sphere of 4000 miles radius, how far at sea can a lighthouse 110 feet high be seen?

20. Determine the sides of an equilateral triangle, having given the lengths of the three perpendiculars drawn from any point within to the sides.

21. Find the number of cubic inches of water that a bowl will hold, whose shape is that of a spherical segment, 10 inches in height, the diameter of the top being 40 inches.

22. Find the side of the largest cube that can be cut from a globe 24 inches in diameter.

23. Which is the greater — 3 solid inches, or 3 inches solid?

24. Three men living 60 miles from one another wish to dig a well that will be the same distance from each of their homes. Where must they dig the well?

25. Bisect a given quadrilateral by a straight line drawn through a vertex.

26. One arm of a right triangle is 30 feet and the perpendicular from the vertex of the right triangle to the hypotenuse is 24 feet. Find the area of the triangle.

27. Three chords, lengths 6, 8, and 10, just go around in a semicircle. Find the radius of the circle.

28. A cone, a half globe, and a cylinder, of the same base and altitude, are as 1 : 2 : 3.

29. Two sides of a triangle are 3 feet and 8 feet, respectively, and inclose an angle of 60°. Find the third side.

30. A rectangular garden is 40 feet by 60 feet. It is surrounded by a road of uniform width, the area of which is equal to the area of the field. Find the width of the road.

31. The sum of the two crescents made by describing semicircles outward on the two sides of a right triangle and a semicircle toward them on the hypotenuse, is equivalent to the right triangle.

32. Prove that the circle through the middle points of the sides of a triangle passes through the feet of the perpendiculars from the opposite vertices, and through the middle points of the segments of the perpendiculars included between their point of intersection and the vertices.

33. What is the volume of the frustum of a sphere, the radius of whose upper base is 3 feet and lower base 4 feet, and altitude 1 foot?

34. If a circle rolls on the inside of a fixed circle of double the radius, find the length of the path that any fixed point in the circumference of the moving circle will trace out.

35. Find the diameter of a circle inscribed in a triangle whose sides are 6, 8, and 10 feet, respectively.

36. Find the diameter of a circle circumscribed about a triangle whose sides are 6, 8, and 10 feet, respectively.

37. What is the area of an equilateral triangle whose sides are 100 inches?

38. What is the area of a tetragon (square) whose sides are 100 inches?

39. What is the area of a regular pentagon whose sides are 100 inches?

40. What is the area of a regular hexagon whose sides are 10 feet?

41. What is the area of a regular heptagon whose sides are 10 feet?

42. What is the area of a regular octagon whose sides are 10 feet?

43. What is the area of a regular nonagon whose sides are 10 feet?

44. What is the area of a regular decagon whose sides are 10 feet?

45. What is the area of a regular undecagon whose sides are 10 feet?

46. What is the area of a regular dodecagon whose sides are 10 feet?

47. Find the side of an inscribed square of a triangle whose base is 10 feet and altitude 4 feet.

48. Find the diameter of a circle of which the height of an arc is 6 inches and the chord of half the arc is 10 inches.

49. Find the height of an arc, when the chord of the arc is 10 inches and the radius of the circle is 8 inches.

50. Find the chord of half an arc, when the chord of the arc is 20 feet and the height of the arc is 2 feet.

51. Find the chord of half an arc, when the chord of the arc is 10 inches and the radius of the circle is 8 inches.

52. Find the side of a circumscribed polygon, when the side of a similar inscribed polygon is 10 feet and the radius of the circle is 30 feet.

53. A log 10 feet long, 2 feet in diameter at one end and 3 feet at the other, is rolled along till the larger end describes a circle. Find the diameter of the circle.

54. At the extremities of the diameter of a circular park stand two electric light posts, one 12 feet high and the other 18 feet high. What points on the circumference of the park are equidistant from the tops of the posts, the diameter of the park being 100 feet?

55. What is the circumference of the largest circular ring that can be put in a cubical box whose edge is 4 feet?

56. What is the side of the largest square that can be inscribed in a semicircle whose diameter is $2\sqrt{5}$ feet?

57. What is the volume of the largest cube that can be inscribed in a hemisphere whose diameter is 3 feet?

58. In a triangle whose base is 30 inches and altitude 18 inches a square is inscribed. Find its area.

59. Two equal circles of 10-inch radii are described so that the center of each is on the circumference of the other. Find the area of the curvilinear figure intercepted between the two circumferences.

60. Two equal circles of 8-inch radii intersect so that their common chord is equal to their radius. Find the area of the curvilinear figure intercepted between the two circumferences.

61. Find the area of a zone whose altitude is 4 feet on a sphere whose radius is 10 feet.

62. Find the volume of a segment of a sphere whose altitude is 1 foot and the radius of the base 2 feet.

63. Mr. Brown has a plank of uniform thickness 10 feet long, 12 inches wide at one end and 5 inches at the other. How far from the large end must it be cut straight across so that the two parts shall be equal?

64. Having given the lesser segment of a straight line divided in extreme and mean ratio, to construct the whole line.

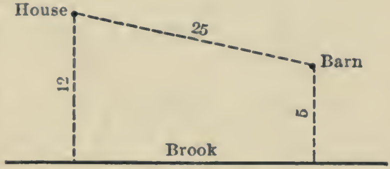
65. Find the volume of a spherical shell whose two surfaces are 64π and 36π.

66. To construct a triangle having given the three medians.

67. Two sides of a quadrilateral lot run east 216 feet and north 63 feet. If the other two sides measure 135 and 180 feet, respectively, what is its area in square yards?

68. If the perimeter of a right triangle is 240 rods and the radius of the inscribed circle 20 rods, what are the sides?

69. On a hillside which slopes 11 feet in 61 feet of its length, stands an upright pole. If this pole should break at a certain point and fall up hill, the top would strike the ground 61 feet from the base of the pole; but if it should fall down hill, its top would strike the ground $48\frac{1}{61}$ feet from the base of the pole. Find the length of the pole.



70. A house and barn are 25 rods apart. The house is 12 rods and the barn 5 rods from a brook running in a straight line. What is the shortest distance one must walk from the house to get a pail of water from the brook and carry it to the barn?

71. Construct geometrically the square root of any number, *n*.

72. Construct a triangle having given the base, the median upon the base, and the difference between the base angles.

73. A man owning a rectangular field 300 feet by 600 feet, wishes to lay out driveways of equal width having the diagonals of the field as center lines, and such that the area of the driveways shall be $\frac{1}{2}$ of the area of the field. Determine the width of the driveways.

74. Two ladders 14 feet apart at their base touch each other at the top. Each is inclined the same, and a round 10 feet up on either side is as far from the top as it is from the base of the other ladder. Get the length of the ladders.

75. A tree 123 feet high breaks off a certain distance up, and the moment the top strikes a stump 15 feet high the broken part points to a spot 108 feet from the base of the tree. Find the length of the part broken off.

76. Divide a triangle into three equivalent parts by lines drawn from a point P within the triangle.

77. From a point P without a circumference, to draw a secant which is bisected by the circumference.

78. To construct a triangle having given the three feet of the altitudes.

79. If from any point in the circumference of a circle perpendiculars be dropped upon the sides of an inscribed triangle (produced, if necessary), the feet of the perpendiculars are in a straight line.

80. Inside a square 10-acre lot a cow was tethered to the fence at a point 1 rod from the corner by a rope just long enough to allow her to graze over an acre of ground. How long was the rope?

81. From any point P in the bisector of the angle A in a triangle ABC, perpendiculars PA', PB', PC' are drawn to the three sides. Prove PA' and $B'C'$ intersect in the median from A.

82. If the bisectors of two angles of a triangle are equal, the triangle is isosceles.

83. In a right triangle the bisector of the right angle also bisects the angle between the perpendicular and the median from the vertex of the right angle to the hypotenuse.

84. Find the locus of a point the sum or the difference of whose distances from two fixed straight lines is given.

85. The bisector of an angle of a triangle is less than half the sum of the sides containing the angle.

86. The difference between the acute angles of a right triangle is equal to the angle between the median and the perpendicular drawn from the vertex of the right angle to the hypotenuse.

87. A hollow rubber ball is 2 inches in diameter and the rubber is $\frac{3}{16}$ inch thick. How much rubber would be used in the manufacture of 1000 such balls?

88. Having given two concentric circles, draw a chord of the larger circle, which shall be divided into three equal parts by the circumference of the smaller circle.

89. The distances from a point to the three nearest corners of a square are 1 inch, 2 inches, and $2\frac{1}{2}$ inches. Construct the square.

90. Draw a chord of given length through a given point, within or without a given circle.

91. Find the greatest segment of a line 10 inches long, when it is divided in extreme and mean ratio.

92. In a quadrilateral $ABCD$, $AB = 10$, $BC = 17$, $CD = 13$, $DA = 20$, and $AC = 21$. Find the diagonal BD.

93. To divide a trapezoid into two similar trapezoids by a line parallel to the base.

94. From a given point in a circumference, to draw a chord that is bisected by a given chord.

95. In a given line AB, to find a point C such that $AC : BC = 1 : \sqrt{2}$.

96. From a given rectangle to cut off a similar rectangle by a line parallel to one of its sides.

97. Find the locus of a point in space the ratio of whose distances from two given points is constant.

98. Find the locus of a point whose distance from a fixed straight line is in a given ratio to its distance from a fixed plane perpendicular to that line.

99. Any point in the bisector of a spherical angle is equally distant from the sides of the angle.

100. If any number of lines in space meet in a point, the feet of the perpendiculars drawn to these lines from another point lie on the surface of a sphere.

101. If the angles at the vertex of a triangular pyramid are right angles, and the lateral edges are equal, prove that the sum of the perpendiculars on the lateral faces from any point in the base is constant.

102. A plane bisecting two opposite edges of a regular tetraedron divides the tetraedron into two equal polyedrons.

103. The volume of a truncated triangular prism is equal to the product of the lower base by the perpendicular on the lower base from the intersection of the medians of the upper base.

104. The point of intersection of the perpendiculars erected at the middle of each side of a triangle, the point of intersection of the three medians, and the point of intersection of the three perpendiculars from the vertices to the opposite sides are in a straight line; and the distance of the first point from the second is half the distance of the second from the third.

105. Three circles are tangent externally at the points A, B, and C, and the chords AB and AC are produced to cut the circle BC at D and E. Prove that DE is a diameter.

106. A cylindrical bucket without a top is 6 inches in circumference and 4 inches high. On the inside of the vessel 1 inch from the top is a drop of honey, and on the opposite side of the vessel 1 inch from the bottom, on the outside, is a fly. How far will the fly have to go to reach the honey?

107. P is any point on the circumcircle of an equilateral triangle ABC; AP, BP meet BC, CA respectively in X, Y. Prove $BX \cdot AY$ is constant.

108. Find the locus of all points from which two unequal circles subtend equal angles.

109. Show that any two perpendicular lines terminated by the opposite sides of a square are equal to one another, and by this property show how to escribe a square to a given quadrilateral.

110. If the incircle passes through the centroid of the triangle, find the relation between the sides a, b, and c.

111. If through a point O within a triangle ABC parallels EF, GH, IK to the sides be drawn, the sum of the rectangles of their segments is equal to the rectangle contained by the segments of any chord of the circumscribing circle passing through O.

112. If two chords intersect at right angles within a circle, the sum of the squares on their segments equals the square on the diameter.

113. If from a point A, without a circle, two secants, ACD and AGK, are drawn, the chords CK and DG intersect on the chord of contact of the tangents from the point A to the circle.

114. If from a given point without a given circle any number of secants are drawn, the chords joining the points of intersection of the secants with the circle all cross on the same straight line.

115. To draw a tangent from a given external point to a given circle by means of a ruler only.

116. Of all polygons constructed with the same given sides, the cyclic polygon is the maximum.

117. The square on the side of a regular inscribed pentagon is equal to the square on the side of a regular inscribed hexagon, plus the square on the side of a regular inscribed decagon.

118. The area of an inscribed regular dodecagon is three times the square of the radius of the circle.

119. The square of the side of an inscribed equilateral triangle is equal to the sum of the squares of the sides of an inscribed square and inscribed regular hexagon.

120. Construct a circumference equal to three times a given circumference.

121. Construct a circle equivalent to three times a given circle.

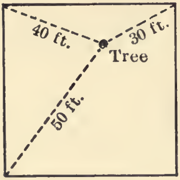
122. If *ABCD* be a cyclic quadrilateral, and if we describe any circle passing through the points *A* and *B*, another through *B* and *C*, a third through *C* and *D*, and a fourth through *D* and *A*; these circles intersect successively in four other points, *E*, *F*, *G*, *H*, forming another cyclic quadrilateral.

123. Construct a triangle, given the altitude, the median, and the angle bisector, all from the same vertex.

124. Prove that the circumcircle of a triangle bisects each of the six segments determined by the incenter and the three excenters of the triangle.

125. If *A*, *B*, *C* are three collinear points, and if *K* is any other point, prove that the circumcenters of the triangles *KBC*, *KCA*, and *KAB* are concyclic with *K*.

126. If the diameter of a circle be divided into any number of segments, and circumferences be described upon these segments as diameters, the sum of these circumferences is equal to the circumference of the original circle.



127. I own a square garden as shown in the above diagram. Within the garden stands a tree 30 feet, 40 feet, and 50 feet respectively from three successive corners. How much land have I?

The Famous Nine-Point Circle.

128. (*a*) If a circle be described about the pedal triangle of any triangle, it will pass through the middle points of the lines drawn from the orthocenter to the vertices of the triangle, and through the middle points of the sides of the triangle, in all, through nine points.

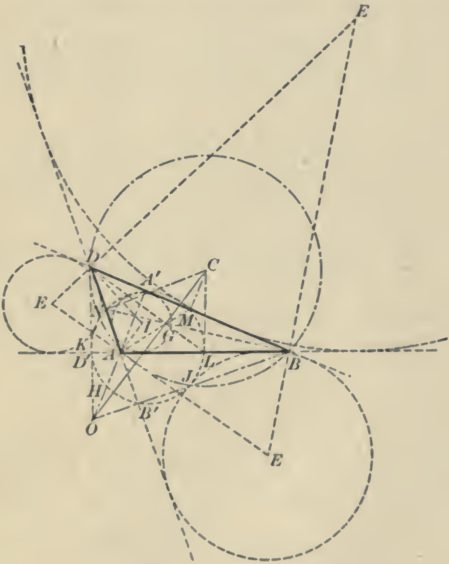
(*b*) The center of the nine-point circle is the middle point of the line joining the orthocenter and the center of the circumcircle of the triangle.

(*c*) The radius of the nine-point circle is half the radius of the circumcircle of the triangle.

(*d*) The centroid of the triangle also lies on the line joining the orthocenter and the center of the circumcircle of the triangle, and divides it in the ratio of 2 : 1.

(*e*) The sides of the pedal triangle intersect the sides of the given triangle in the radical axis of the circumscribing and nine-point circles.

(*f*) The nine-point circle is tangent to the inscribed and escribed circles of the triangle.



Let ABD be any triangle; A', B', D', the projections of the vertices on the opposite sides; H, J, K, the mid-points of OA, *OB*, *OD*, respectively, *O* being the orthocenter. Let *L*, *M*, *N* be the mid-points of the sides. Join *F*, *E*, and *D*. The △ *A′B′D′* is called the pedal triangle. The nine points *A′*, *N*, *K*, *D′*, *H*, *B′*, *J*, *L*, *M* are concyclic; and the circle through them is the nine-point circle of the triangle.

For the proofs of these theorems, see "Finkel's Mathematical Solution Book" and the monograph, "Some Noteworthy Properties of the Triangle and Its Circles," by Dr. W. H. Bruce, president of the North Texas State Normal School, Denton.

129. If from any point in either side of a right triangle, a line is drawn perpendicular to the hypotenuse, the product of the segments of the hypotenuse is equal to the product of the segments of the side plus the square of the perpendicular.

130. *A*, *B*, and *C* are fixed points. Describe a square with one vertex at *A*, so that the sides opposite to *A* pass through *B* and *C*.

131. If *ABCD* is a cyclic quadrilateral, prove that the centers of the circles inscribed in triangles *ABC*, *BCD*, *CDA*, *DAB* are the vertices of a rectangle.

132. A round hole one foot in diameter is cut through a sphere 20 inches in diameter. Find the volume of the part remaining, the axes of the hole passing through the center of the sphere.

133. Given the incenter, circumcenter, and one excenter of a triangle, construct it.

134. Divide the triangle whose sides are 7, 15, 20 into two equivalent parts by a radius of the circumcircle.

135. Construct a triangle, given its altitude and the radii of the inscribed and circumscribed circles.

136. In the semicircle *ABCD* express the diameter *AD* in terms of the chords *AB*, *BC*, and *CD*.

137. On one side of an equilateral triangle describe outwardly a semicircle. Trisect the arc and join the points of division with the vertex of the triangle. Find the ratio of the segments of the diameter.

138. If a, b, c are the sides of a triangle, and $5\,(a^2+b^2+c^2) = 6\,(ab+bc+ac)$, show that the incircle passes through the centroid of the triangle.

139. If through the vertices of any inscribed polygon tangents are drawn forming a circumscribed polygon, the continued product of the perpendiculars from any point in the circle on the sides of the inscribed polygon is equal to the continued product of the perpendiculars from the same point on the sides of the circumscribed polygon.

140. A lot 100 feet long and 60 feet wide has a walk extending from one corner halfway around it, and occupying one third of the area. Required the width of the walk. A geometrical construction is desired.

141. Construct a triangle, having given the vertical angle, the sum of the three sides, and the perpendicular.

142. Prove that the dihedral angle of a regular octahedron is the supplement of the dihedral angle of a regular tetrahedron.

143. Given the three diagonals of an inscriptible quadrilateral, to construct the quadrilateral.

144. P is a point on the minor arc AB of the circumcircle of the regular hexagon $ABCDEF$; prove that $PE + PD = PA + PB + PC + PF$.

145. In a right triangle the hypotenuse is 17 and the diameter of the inscribed circle 6. Another equal circle is described touching the base produced and the hypotenuse; how far apart are the centers of the two circles?

146. Two equal circular discs are to be cut out of a rectangular piece of paper, 9 inches long and 8 inches wide. What is the greatest possible diameter of the discs?

MISCELLANEOUS PROBLEMS

1. A seed is planted. Suppose at the end of 2 years it produces a seed, and one each year thereafter; each of these when 2 years old produces a seed yearly. All the seeds produced do likewise. How many seeds will be produced in 20 years?

2. If a 4-inch auger hole be bored diagonally through a 12-inch cube, what will be the volume displaced, the axis of the auger hole coinciding with the diagonal of the cube?

3. I have a circular orchard 110 yards in diameter. How many trees can be set in it so that no two shall be within 16 feet of each other, and no tree within 5 feet of the fence?

4. What is the convex surface and volume of a cylindric ungula whose least length is 5 feet, greatest length 13 feet, the radius of the base being $1\frac{1}{2}$ feet?

5. What is the length of the arc whose chord is 16 feet and height 6 feet?

6. Find the area of a sector, having given the chord of the arc equal to 16 feet, and the height of the arc equal to 6 feet.

7. What is the area of a segment whose base is 6 feet and height 2 feet?

8. Find the volume of an iron rod 2 inches in diameter and 10 feet from end to end containing a loop whose inner diameter is 4 inches.

9. What is the area of a circular zone, one side of which is 30 inches and the other 40 inches, and the distance between them 10 inches?

10. The shell of a hollow iron ball is 4 inches thick, and contains $\frac{1}{5}$ of the number of cubic inches in the whole ball. Find the diameter of the ball.

11. A rope 60 feet long wraps around two trees 6 feet and 10 feet in diameter, respectively, and crosses between them. Find the distance between their centers.

12. On the tire of a wheel 4 feet in diameter is a black spot. How far does the spot move while the wheel makes 4 revolutions?

13. A fly lights on the spoke of a carriage wheel 4 feet in diameter, 1 foot up from the ground. How far will the fly have traveled when the wheel has made 2 revolutions on a level plane?

14. An eagle and a sparrow are in the air; the eagle is 100 feet above the sparrow. If the sparrow flies straight forward in a horizontal line, and the eagle flies twice as fast directly towards the sparrow, how far will each fly before the sparrow is caught?

15. A cow is tethered to the corner of a barn 25 feet square, by a rope 100 feet long. How many square feet can she graze?

16. A solid cube weighs 300 pounds. If a power is applied at an angle of 45° at an upper edge of the cube, how many foot pounds will be required to overturn the cube?

17. A tree 110 feet high, standing by the side of a stream 100 feet wide, is broken by a storm; the fallen part is undetached from the stump, and its top rests 10 feet above the water and points directly to the opposite shore. How high is the stump?

18. At the edge of a circular lake 1 acre in area stands a tree. What length of rope, tied to this tree, will allow a horse to graze upon $\frac{1}{4}$ of an acre?

19. A horse is tied to a stake in the circumference of a 6-acre field. How long must the rope be to allow him to graze over just 1 acre inside the field?

20. What is the longest piece of carpet 3 feet wide, cut square at the ends, that can be put in a room 16 feet by 20 feet?

21. The fore wheel and the hind wheel of a carriage are 12 feet and 15 feet in circumference, respectively; a rivet in the tire of each is observed to be up when the carriage starts. How far will each rivet have moved when they are next up together?

22. A log 40 inches in diameter is to be sawed by four men. What part of the diameter must each man saw to do $\frac{1}{4}$ of the work?

23. What is the length of a chord cutting off the fourth part of a circle whose radius is 10 feet?

24. Find the length of a chord cutting off the third part of a circle whose diameter is 40 feet.

25. A tree 80 feet high was broken in a storm so that the top struck the ground 40 feet from the foot of the tree. If the tree remained in contact, what was the length of the path through which the top of the tree passed in falling to the ground?

26. By boring through the center of a wooden ball, with an auger 4 inches in diameter, $\frac{1}{6}$ of the solid contents of the ball is displaced. Find the diameter of the ball.

27. Find the diameter of an auger that will displace $\frac{1}{5}$ of the solid contents of a ball 5 feet in diameter, by boring through its center.

28. Three horses are tethered each to a rope 42 feet in length to the corners of an equilateral triangle whose side is 80 feet. Over how many square feet can each graze, provided they are at no time upon the same ground?

29. How many acres of water can a man see, standing on a ship, with his eyes just 14 feet above the water, when there is no land in sight?

30. In a farmer's pasture is located a triangular house, the length of each side being 10 yards. The farmer wishing to graze his horse finds that stakes are not plentiful and decides to tie the rope to one corner of the house. If the rope is long enough to allow the horse to graze 30 yards from the corner of the house, over how much ground can the horse graze?

31. Three men wish to carry each $\frac{1}{3}$ of an 8-foot log of uniform size and density. Where must the hand stick be placed so that the one at the end of the log and the others at the ends of the stick shall each carry equal weights?

32. If three equal circles are tangent to each other, each to each, and inclose a space between the three arcs equal to 100 square inches, find their radius.

33. If three equal circles are tangent to each other, each to each, with a radius of 10 inches, find the area of the space inclosed between the three arcs.

34. If 4 acres pasture 40 sheep 4 weeks, and 8 acres pasture 56 sheep 10 weeks, how many sheep will 20 acres pasture 50 weeks, the grass growing uniformly all the time?

35. A rabbit 60 yards due east of a hound is running due south 20 feet per second; the hound gives chase at the rate of 25 feet per second. How far will each run before the rabbit is caught?

36. How many fruit trees can be set out upon a space 100 feet square, allowing no two to be nearer each other than 10 feet?

37. How many stakes can be driven down upon a space 12 feet square, allowing no two to be nearer each other than 1 foot?

38. The sum of the sides of a triangle is 100. The angle at A is double that at B, and the angle at B is double that at C. Find the sides.

39. A conical glass 4 inches in diameter and 6 inches in altitude, is filled with water. How much water will run out if it be turned through an angle of 45°?

40. At what latitude is the circumference of a parallel half that of the equator, regarding the earth a perfect sphere?

41. The difference between the circumscribed and inscribed squares of a circle is 72. What is the area of the circle?

42. A drawer made of inch boards is 8 inches wide, 6 inches deep, and slides horizontally. How far must it be drawn out to put into it a book 4 inches thick, 6 inches wide, and 9 inches long?

43. With what velocity must a pail of water be whirled over the head to prevent the water from falling out, the radius of the circle of revolution being 4 feet?

44. Two hunters killed a deer, and wishing to ascertain its weight they placed a rail across a fence so that it balanced with one on each end. They then exchanged places, and the lighter man taking the deer in his lap, the rail again balanced. Find the weight of the deer, the hunters' weights being 160 and 200 pounds.

45. At each corner of a square pasture whose sides are 100 feet a cow is tied with a rope 100 feet long. What is the area of the part common to the four cows?

46. Find the volume generated by the revolution of a circle 10 feet in diameter about a tangent.

47. Find the volume generated by revolving a semicircle 20 inches in diameter about a tangent parallel to its diameter.

48. A circle of 10 inches radius, with an inscribed regular hexagon, revolves about an axis of rotation 20 inches distant from its center and parallel to a side of the hexagon. Find the difference in area of the generated surfaces.

49. Find the difference in the volumes of the two generated solids.

50. An equilateral triangle rotates about an axis without it, parallel to, and at a distance 10 inches from one of its sides. Find the surface thus generated, a side of the triangle being 4 inches.

51. A rectangle whose sides are 6 inches and 18 inches is revolved about an axis through one of its vertices, and parallel to a diagonal. Find the surface thus generated.

52. Find the surface of a square ring described by a square foot revolving round an axis parallel to one of its sides and 4 feet distant.

53. Find the volume generated by an ellipse whose axes are 40 inches and 60 inches, revolving about an axis in its own plane whose distance from the center of the ellipse is 100 inches.

54. What power acting horizontally at the center of a wheel $4\frac{1}{2}$ feet in diameter and weighing 270 pounds, will draw it over a cylindrical log 6 inches in diameter, lying on a horizontal plane?

55. Find the volume generated by the revolution of a circle 2 feet in diameter about a tangent.

56. Find the surface generated by the revolution of a circle 2 feet in diameter about a tangent.

57. Find the surface and volume of a cylindric ring, the diameter of the inner circumference being 12 inches and the diameter of the cross section 16 inches.

58. Find the surface and volume of the segment of the same cylindric ring, if a plane is passed perpendicular to its axis, and at a distance of 4 inches from the center.

59. A galvanized cistern is 8 feet in diameter at the top, 10 feet at the bottom, and 10 feet deep. A plane passes from the top on one side to the bottom on the other side. What is the volume of the part contained between this plane and the base?

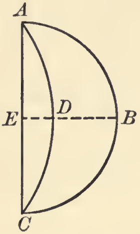
60. A wineglass in the form of a frustum of a cone is 4 inches in diameter at the top, 2 inches at the bottom, and 5 inches deep. If, when full of water, it is tipped just so that the raised edge at the bottom is visible, what is the volume of the water remaining in the glass?

61. To what depth will a sphere of cork, 2 feet in diameter, sink in water, the specific gravity of cork being .25?

62. The diameter of two equal circular cylinders, intersecting at right angles, is 3 feet. What is the surface common to both?

63. In digging a well 4 feet in diameter, I come to a log 4 feet in diameter lying directly across the entire well. What was the contents of the part of the log removed?

64. What is the volume of a solid formed by two cylindric rings 2 inches in diameter, whose axes intersect at right angles and whose inner diameters are 10 inches?



65. Find the area of a circular lune or crescent $ABCD$; the chord $AC = 10$ feet; the height $EB = 3$ feet; and the height $ED = 2$ feet.

66. Find the circumference of an ellipse, the transverse and conjugate diameters being 80 inches and 30 inches.

67. The axes of an ellipse are 60 inches and 20 inches. What is the difference in area between the ellipse and a circle having a diameter equal to the conjugate axis?

68. What is the area of a parabola whose base, or double ordinate, is 30 inches and whose altitude, or height, is 20 inches?

69. What is the area of a cycloid generated by a circle whose radius is 6 feet?

70. Two men, A and B, started from the same point at the same time; A traveled southeast for 10 hours, and at the rate of 10 miles per hour, and B traveled due south for the same time, going 6 miles per hour; they turned and traveled directly towards each other at the same rates respectively, till they met. How far did each man travel?

71. In front of a house stand two pine trees of unequal height; from the bottom of the second to the top of the first a rope 80 feet in length is stretched, and from the bottom of the first to the top of the second a rope 100 feet in length is stretched. If these ropes cross 10 feet above the ground, find the distance between the trees.

72. To trisect any angle.

73. A grocer has a platform balance the ratio of whose arms is 9 to 10. If he sells 20 pounds of merchandise to one man, weighing it on the right-hand pan, and 20 pounds to another man, weighing it on the left-hand pan, what per cent does he gain or lose by the two transactions?

74. A and B carry a fish weighing 54 pounds hung between them from the middle of a 10-foot oar. One end of the oar rests on A's shoulder, but the other end is pushed 1 foot beyond B's shoulder. What part of the weight does each carry?

75. A half-ounce bullet is fired with a velocity of 1400 feet per second from a gun weighing 7 pounds. Find the velocity in feet per second with which the gun begins to recoil, and the mean force in pounds' weight that must be exerted to bring it to rest in 4 inches.

76. A bullet fired with a velocity of 1000 feet per second penetrates a block of wood to a depth of 12 inches. If it were fired through a plank of the same wood, 2 inches thick, what would be its velocity on emergence, assuming the resistance of the wood to the bullet to be constant?

77.* A horse is tied to one corner of a rectangular barn 30 by 40 feet. What is the surface over which the horse can range if the rope with which he is tied is 80 feet long?

78.* How many acres are there in a circular tract of land, containing as many acres as there are boards in the fence inclosing it, the fence being 5 boards high, the boards 8 feet long, and bending to the arc of a circle?

79.* A thread passes spirally around a cylinder 10 feet high and 1 foot in diameter. How far will a mouse travel in unwinding the thread if the distance between the coils is 1 foot?

80. A string is wound spirally 100 times around a cone 100 feet in diameter at the base. Through what distance will a duck swim in unwinding the string, keeping it taut at all times, the cone standing on its base at right angles to the surface of the water?

81.* After making a circular excavation 10 feet deep and 6 feet in diameter, it was found necessary to move the center 3 feet to one side, the new excavation being made in the form of a right cone having its base 6 feet in diameter and its apex in the surface of the ground. Required the total amount of earth removed.

82.* A 20-foot pole stands plump against a perpendicular wall. A cat starts to climb the pole, but for each foot it ascends, the pole slides one foot from the wall; so that when the top of the pole is reached, the pole is on the ground at right angles to the wall. Required the distance through which the cat moved.

*** These problems are from "Finkel's Solution Book."**

83. A tree 96 feet high was broken by the wind in such a manner that the top struck the ground 36 feet from the foot of the tree. If the parts remained connected at the place of breaking, forming with the ground a right triangle, how high was the stump?

84. The distance around a rectangular field is 140 rods, and the diagonal is 50 rods. Find its length, breadth, and area.

85. The area of a rectangular field is 30 acres, and its diagonal is 100 rods. Find its length and breadth.

86. Two trees of equal height stand upon the same level plane, 60 feet apart and perpendicular to the plane. One of them is broken off close to the ground by the wind, and in falling it lodges against the other tree, its top striking 20 feet below the top of the other. Find the height of the trees.

87. A square field contains 10 acres. From a point in one side, 10 rods from the corner, a line is drawn to the opposite side cutting off $6\frac{1}{4}$ acres. How long is the line?

88. Find the edge of the largest hollow cube, having the shell three inches in thickness, that can be made from a board $42\frac{1}{4}$ feet long, 2 feet wide, and 3 inches thick.

89. A circular farm has two roads crossing it at right angles 40 rods from the center, the roads being 60 and 70 rods respectively, within the limits of the farm. Find the area of the farm.

90. The longest straight line that can be stretched in a circular track is 200 feet in length. Find the area of the track.

91. From the two acute angles of a right triangle lines are drawn to the middle points of the opposite sides; their respective lengths are $\sqrt{73}$ and $\sqrt{52}$ feet. Find the sides of the triangle.

92. A wheel of uniform thickness, 4 feet in diameter, stands in the mud 1 foot deep. What fraction of the wheel is out of the mud?

MATHEMATICAL RECREATIONS

1. Mary is 24 years old. She is twice as old as Ann was when Mary was as old as Ann is now. How old is Ann?

2. There is a great big turkey that weighs 10 pounds and a half of its weight besides. What is its weight?

3. With 6 matches form 4 equilateral triangles, the side of each being equal to the length of a match.

4. One tumbler is half full of wine, another is half full of water. From the first tumbler a teaspoonful of wine is taken out and poured into the tumbler containing the water. A teaspoonful of the mixture in the second tumbler is then transferred to the first tumbler. As the result of this double transaction is the quantity of wine removed from the first tumbler greater or less than the quantity of water removed from the second tumbler?

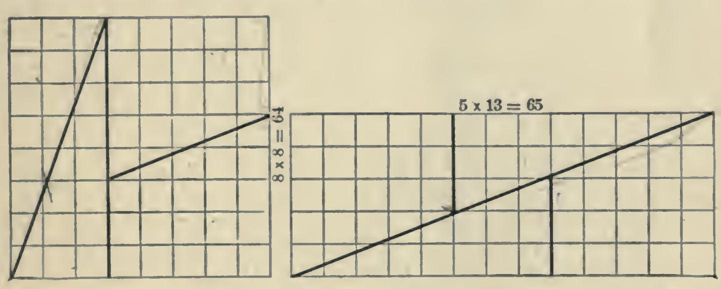
5. (i) Take any number; (ii) reverse the digits; (iii) find the difference between the number formed in (ii) and the given number; (iv) multiply this difference by any number you please; (v) cross out any digit except a naught; (vi) give me the sum of the remaining digits, and I will give you the figure struck out.

6. (i) Take any number; (ii) add the digits; (iii) subtract the sum of the digits from the given number; (iv) cross out any digit except a naught; (v) give me the sum of the remaining digits, and I will give you the figure struck out.

7. Given a plank 12 inches square, required to cover a hole in a floor 9 inches by 16 inches, cutting the plank into only two pieces.

8. Place four 9's in such a manner that they will exactly equal 100.

9. The square is 8 inches by 8 inches. By forming the latter figure out of the four parts of the square it is found to be



5 inches by 13 inches and contains 65 square inches. Where does the other inch come from?

10. A teamster brought 5 pieces of chain of 3 links each to a blacksmith, and asked the cost of making them into one piece of chain. The blacksmith replied, "I charge 2 cents to cut a link and 2 cents to weld a link." The teamster remarked that as it would require 4 cuts and 4 welds the charge would be 16 cents. "No, you are mistaken," said the blacksmith, "I figure it but 12 cents." Who was right?

11. THE HARE AND THE HOUND

A hare is 10 rods before a hound, and the hound can run 10 rods while the hare runs 1 rod. Prove that the hound will never catch the hare.

Proof. — When the hound runs 10 rods the hare has gone 1 rod. When the hound goes the 1 rod the hare has run $\frac{1}{10}$ of a rod, and when the hound has run the $\frac{1}{10}$ of a rod the hare has run $\frac{1}{100}$ of a rod, and so on. Therefore, the hare is always a fraction of a rod ahead of the hound, and hence the hound will never catch the hare.

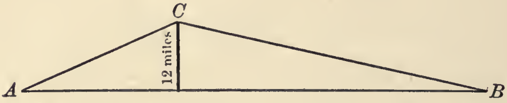
12. To prove that 1 equals 2.

Let $x = 1.$
Then $x^2 = x.$
$x^2 - 1 = x - 1.$
Factoring, $(x + 1)(x - 1) = x - 1.$
Dividing, $x + 1 = 1.$
But $x = 1.$ Therefore $1 = 2.$

13. A Young Lady to Her Lover—

I ask you, sir, to plant a grove
  To show that I'm your lady love.
This grove though small must be composed
  Of twenty-five trees in twelve straight rows.
In each row five trees you must place
  Or you shall never see my face.

14. In going from *A* to *B*, through mistake I take the road going via *C*, which is nearer *A* than *B* and is 12 miles to the left of the road I should have traveled. After reaching *B* I find that I have traveled 35 miles. Find the distances from *A* to *B*, *A* to *C*, and *C* to *B*, each being an integer.



15. A room is 30 feet long, 12 feet wide, and 12 feet high. On the middle line of one of the smaller side walls and 1 foot from the ceiling is a fly. On the middle line of the opposite wall and 1 foot from the floor is a spider. The fly being paralyzed by fear remains still until the spider catches it by crawling the shortest route. How far did the spider crawl?

16. A train 1 mile long starts from the station at Glady. The engine leaves the station and the conductor waits until the caboose comes, when he jumps on the caboose and walks forward over the train. When the engine reaches the next station, Oxley, 4 miles distant from Glady, the conductor steps off the engine. How far does the conductor ride and how far does he walk?

17. ZENO'S PARADOXES ON MOTION

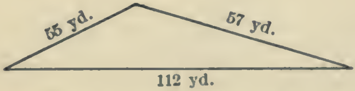
(*a*) Since an arrow cannot move where it is not, and since also it cannot move where it is (in the space it exactly fills), it follows that it cannot move at all.

(*b*) The idea of motion is inconceivable, for what moves must reach the middle of its course before it reaches the end. Hence the assumption of motion presupposes another motion, and that in turn another, and so ad infinitum.

18. I have only $2 when approached by a friend whom I owe $2. The friend asks for what I owe him, so I give him the $2 and remark that it is all my money. My friend sympathizing with me in my poverty, hands me back a dollar and says, "I will mark your account paid." What per cent did I gain by the transaction?

19. WHAT WERE OUR AGES WHEN MARRIED?

When first the marriage knot was tied between my wife and me,
Her age did mine as far exceed, as three plus three does three;
But when three years and half three years we man and wife had been,
Our ages were in ratio then as twelve is to thirteen.



20. Find the value of the above lot at $1 per square yard.

21. How much dirt is there in a hole the dimensions of which are an inch?

22. Which is correct to say, Five and six are twelve, or to say, Five and six is twelve?

23. Three men, A, B, and C, wish to divide $60 among themselves so as to receive a third, fourth, and fifth, respectively. How much should each receive?

24. A, B, and C are in partnership. They own 17 sheep. They wish to divide them, — one to get $\frac{1}{2}$, one to get $\frac{1}{3}$, and the other to get $\frac{1}{9}$. How can this be done without killing a sheep?

25. If 6 cats eat 6 rats in 6 minutes, how many cats will it take to eat 100 rats in 100 minutes?

26. A man who owned a piece of land in the form of a square, decided to divide it among his wife and four sons, so as to give his wife $\frac{1}{4}$ in the shape of a square in one corner and to give the remaining $\frac{3}{4}$ to his sons. He divided the land so that each son received the same amount of land and the four pieces were similar. How did he divide it?

27. A philosopher had a window a yard square, and it let in too much light. He blocked up one half of it, and still had a square window a yard high and a yard wide. Show how he did it.

28. Why does it take no more pickets to build a fence down a hill and up another than in a straight line from top to top, no matter how deep the gully?

29. A room with eight corners had a cat in each corner, seven cats before each cat, and a cat on every cat's tail. How many cats were in the room?

30. (i) Take any number of three unequal digits; (ii) reverse the order of the digits; (iii) subtract the number so formed from the original number; (iv) give me the last digit of the difference, and I will give you the difference.

31. Select any two numbers, each of which is less than 10. (i) choose either of them and multiply it by 5; (ii) add 7 to the result; (iii) double this result; (iv) to this add the other number; (v) give me the result, and I will give you the numbers originally selected, and also tell you which one you multiplied by 5.

32. (i) Take any number of three unequal digits, in which the first and last differ by not less than 2; (ii) form a new number by reversing the order of the digits; (iii) take the difference between these two numbers; (iv) form another number by reversing the order of the digits in this difference; find the sum of the results in (iii) and (iv). The sum will be 1089.

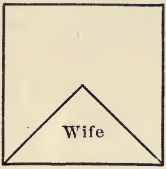
33. Write down a number of three or more figures, divide by 9, and name the remainder; erase one figure of the number, divide by 9, and tell me the remainder, and I will tell you what figure you erased.

34. Let a person write down a number greater than 1 and not exceeding 10; to this I will add a number not exceeding 10, alternately with him; and, although he has the advantage in putting down the first number, I will reach the even hundred first.

35. A boy bought a pair of boots for $2 and gave a $10 bill in payment. The merchant had a friend change the bill, and gave the boy his change. The boy left the city with the boots and the $8. The friend returned the bill, saying it was a counterfeit, and the merchant had to give him good money for it. What was the merchant's loss?

36. A man having a fox, a goose, and a peck of corn is desirous of crossing a river. He can take but one at a time. The fox will kill the goose and the goose will eat the corn if they are left together. How can he get them safely across?

37. Suppose a hole to be cut through the earth, and a ball dropped into this hole, what would be the behavior of the ball, and where would it come to rest and how?



38. A man died leaving his wife and four children a piece of land as shown in the figure. The wife is to have $\frac{1}{4}$ in the shape of a triangle. The children's parts are to be similar, and equal in size. How must the land be divided?

39. With what four weights can you weigh any number of pounds from 1 to 40?

40. Can you plant 19 trees in 9 rows with 5 trees to the row?

41. Do figures ever lie?

42. Can you multiply feet by feet and get square feet?

43. A hunter walked around a tree to kill a squirrel; the squirrel kept behind the tree from the hunter. Did he go around the squirrel?

44. A FALLACY.

Let x be a quantity which satisfies the equation

$$e^x = -1.$$

Squaring both sides, $e^{2x} = 1$.

$$\therefore 2x = 0.$$

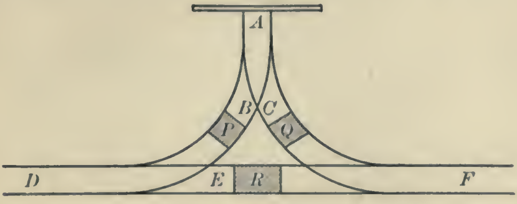
$$\therefore x = 0.$$

$$\therefore e^x = e^0.$$

But $e^x = -1$ and $e^0 = 1$. $\therefore -1 = 1$.

45. I have $10,000. If I spend half of this sum to-day and half of the remainder each day following, in how many days will I have no money?

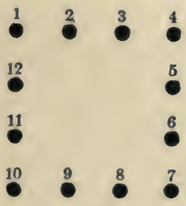
46. In the diagram, DEF is a railroad with two sidings, DBA and FCA, connected at A. The portion of the rails at A which is common to the two sidings is long enough to permit of a single car like P or Q, running in or out of it; but it is too



short to contain the whole of an engine like *R*. Hence if an engine runs up one siding, such as *DBA*, it must come back the same way.

Car No. 1 is placed at *B*, car No. 2 is placed at *C*, and an engine is placed at *E*.

By the use of the engine interchange the cars, without allowing any flying shunts.

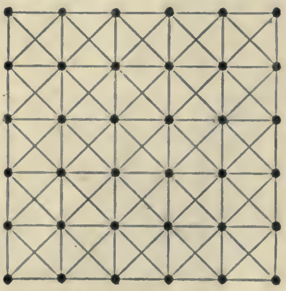


47. Given twelve coins arranged as in the figure. Can you move them so as to have five on a side instead of four, not being allowed to introduce other coins or to destroy the given square?

48. *A* and *B* have an 8-gallon cask of wine and wish to divide it into two equal parts. The only measures they have are a 5-gallon cask and a 3-gallon cask. How can they divide it?

49. I bought a horse for $90, sold it for $100, and soon repurchased it for $80. How much did I make by trading?

50. Stick six pins in the dots so that no two are connected by a straight line.



51. Let x and y be two unequal numbers, and let z be their arithmetical mean.

Then,
$$x + y = 2z.$$
$$\therefore (x + y)(x - y) = 2z(x - y).$$
$$\therefore x^2 - y^2 = 2xz - 2yz.$$
$$\therefore x^2 - 2xz = y^2 - 2yz.$$
$$\therefore x^2 - 2xz + z^2 = y^2 - 2yz + z^2.$$
$$\therefore (x - z)^2 = (y - z)^2.$$
$$\therefore x - z = y - z.$$
$$\therefore x = y.$$

52. To prove $-1 = 1$.

First solution:
$$\frac{-a^{\frac{1}{2}}}{a^{\frac{1}{2}}} = \frac{a^{\frac{1}{2}}}{-a^{\frac{1}{2}}}.$$
$$\therefore (-a^{\frac{1}{2}})^2 = (a^{\frac{1}{2}})^2.$$
$$\therefore -a = a.$$
$$\therefore -1 = 1.$$

Second solution:
$$(-1)^2 = 1.$$
$$\therefore 2 \log(-1) = \log 1 = 0.$$
$$\therefore -1 = e^0.$$

But $e^0 = 1$. $\therefore -1 = 1$.

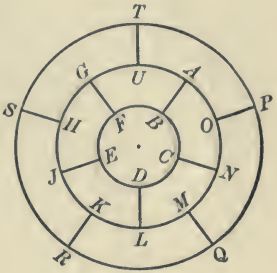
53. With the seven digits, 9, 8, 7, 6, 5, 4, 0, express three numbers whose sum is 82, each digit being used only once, and the use of the usual notations for fractions being allowed.

54. With the ten digits, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0, express numbers whose sum is unity, each digit being used only once.

55. With the nine digits, 9, 8, 7, 6, 5, 4, 3, 2, 1, express four numbers whose sum is 100, each digit being used only once.

56. With the ten digits, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0, express zero, each digit being used only once.

57. With the ten digits, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0, express three numbers whose sum is $2\frac{1}{2}$, each digit being used only once.



58. In the accompanying diagram the letters stand for various towns and the lines indicate the only possible paths by which a person may travel. Show how a person may start from any town and go to every other town once, and only once, and return to the initial town.

59. An oar is a lever of what class?

60. A man hires a livery team to drive from A to C via B and return for $3. At B, midway between A and C, he takes a passenger to C and back to B. What should he charge the passenger?

61. Put down the figures from 1 to 9, leaving out the 8, thus:

1 2 3 4 5 6 7 9

Select any one of the figures, multiply it by 9, then multiply the whole row by that product. Tell me what your answer is, and I will tell you what number you selected.

62. Say to one person:

"Think of a number less than 10; double it; add 16; divide by 2; subtract your first number, and your answer is 8."

Say to another:

"Think of a number less than 10; double it; add 9; divide by 2; subtract your first number, and your answer is $4\frac{1}{2}$."

You can go on indefinitely, giving these mental exercises, no two alike, to each one in a large audience, and announce the answer as quickly as they get it themselves. The secret is this: the final answer is always half the number you tell them to add.

63. If a hen and a half laid an egg and a half in a day and a half, how many eggs would 7 hens lay at the same rate in 6 days?

64. What is the shortest distance that a fly will have to go, crawling from one of the lower corners of a room to the opposite upper corner, the room being 20 feet long, 15 feet wide, and 10 feet high?

65. If a man charges $2 for sawing a cord of wood 3 feet long into 3 pieces, what should he charge for sawing a cord of wood 6 feet long into pieces the same length?

66. Three boys having 10, 30, and 50 apples visit a city and sell them at the same rate and receive the same amount for them. How much do they receive for the apples and at what rate do they sell them?

67. When a boy see-saws on the long end of a plank he balances against 16 bricks, but if he sits on the shorter arm of the plank and places the bricks on the other end he balances against just 11. Find the boy's weight if a brick weighs equal to a three-quarter brick and three quarters of a pound.

68. A switch to a single-track railroad is just long enough to clear a train of 19 cars and a locomotive. How can two trains of 19 cars and a locomotive each, going in opposite directions, pass each other, if a third train of equal length stands on the switch, without dividing a train?

69. A boy was sent to a spring with a 5 and a 3 quart measure to procure exactly 4 quarts of water. How did he measure it?

70. What is the greatest number which will divide 27, 48, 90, and 174 and leave the same remainder in each case?

71. There is in the floor of a granary a hole 2 feet in width and 15 feet in length. How can it be entirely covered with a board 3 feet wide and 10 feet long, by cutting the board only once?

72. What part of ½ square yard is ½ yard square?

73. Can you take 1 from 19 and get 20?

74. If an egg weighs 8 ounces and half an egg, what does an egg and a half weigh?

75. How would you arrange the figures 8, 6, and 1 so that the whole number formed will be divisible by 6?

76. What three figures multiplied by 4 will make precisely 5?

77. Mr. Jackson owns a square farm the area of which is 20 acres; near each corner stands a large tree which is upon a neighbor's land. How may he add to his farm so as to have a square farm containing 40 acres and still not own the land upon which the trees stand?

78. A gentleman rented a farm, and contracted to give a landlord ⅖ of the produce; but prior to the dividing of the corn, the tenant used 45 bushels. When the general division was made, it was proposed to give to the landlord 18 bushels of the heap, in lieu of his share of the 45 bushels which the tenant had used, and then to begin and divide the remainder as though none had been used. Would the method have been correct?

79. What is the difference between half a dozen dozen, and six dozen dozen?

80. What is the difference between twice twenty-five and twice five and twenty?

81. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 0 = ?$

82. If you were required to sell apples by the cubic inch, how would you find the exact number of cubic inches in a dozen dozen?

83. A man who has only two rows of corn hires A and B to hoe them. A hoes three hills on B's row and then begins on his own row. B finishes his row and hoes six hills on A's row, when they find the work is finished. Which man hoes the more and how much more, the rows containing the same number of hills?

84. Two ducks before a duck, two ducks behind a duck, and a duck in the middle, are how many ducks?

85. Can you write 30 with 3 equal figures?

86. Add 1 to 9 and make it 20.

87. Twenty-one ears of corn are in a hollow stump. How long will it take a squirrel to carry them all out if he carries out 3 ears a day?

88. In the bottom of a well 45 feet in depth there was a frog who commenced traveling toward the top. In his journey he ascended 3 feet every day, but fell back 2 feet every night. In how many days did he get out of the well?

89. How many quarter-inch blocks will it take to fill an inch hole?

90. Cut a piece of cardboard 12½ inches long by 2 inches wide into 4 pieces in such a manner as to form a perfect square, without waste.

91. A man and his wife, each weighing 150 pounds, with two sons, each weighing 75 pounds, have to cross a river in a boat which is capable of carrying only 150 pounds' weight. How will they get across?

92. Two men laid a wager as to which could eat the more oysters; one ate ninety-nine, and the other a hundred and won. How many did both together eat?

93. Thrice naught is naught, what is the third of infinity?

94. If $\frac{1}{4}$ of 20 is 4, what will $\frac{1}{3}$ of 10 be?

95. If the third of 6 be 3, what must the fourth of 20 be?

96. Write 24 with 3 equal figures, neither of them being 8.

97. If you cut 30 yards of cloth into one-yard pieces, and cut 1 yard every day, how long will it take?

98. What number is that when multiplied by 18, 27, 36, 45, 54, 63, 72, 81, and 99 gives a product in which the first and last figures are the same as those in the multiplier, and when multiplied by 9, and 90, gives a product in which the last figures are the same as those of the multiplier?

99. Three market women, having severally 10, 30, and 50 oranges, sold them at the same rate, and received the same amount of money. What were the rates and the amounts each received?

100. Suppose a steamer in rapid motion and on its deck a man jumping. Can he jump farther by leaping the way the boat is moving, or in the opposite direction?

101. After killing a certain number of cattle, it was found that twenty fore feet remained. How many head were killed?

102. Can you write 27 with two equal figures?

103. When is a number divisible by 9?

104. Find the figure that may be placed anywhere in, or before, or after, the number 302,011, and make it divisible by 9.

105. In a lot where there are some horses and grooms, can be counted 82 feet and 26 heads. How many horses and grooms are in the lot?

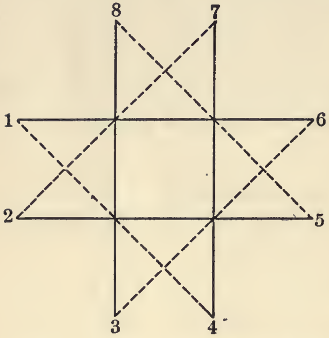
106. If a herring and a half cost a penny and a half, how much will 11 herring cost?

107. What number is it when divided by 2, 3, 4, 5, or 6, there is a remainder of 1, but when divided by 7, there is no remainder?

108. A cord passing over a pulley hung to a pair of cotton scales, suspended from a beam, has a 150-pound weight fastened to one end and the other fastened to an immovable iron stake. How much will the scales register? How much more will they register if a 100-pound weight is hung to a loop in the cord halfway between the pulley and the stake?

109. Why can a fat man swim more easily than a lean one?

110. A rifle ball thrown against a board standing edgewise will knock it down; the same bullet fired at the board will pass through it without disturbing its position. Why is this?



111. Can you mark seven numbers by moving on a straight line from one number to another, as in the figure, marking the number you move to? Do not start twice from the same number.

112. The sum of four figures in value will be
About seven thousand nine hundred and three;
But when they are halved, you'll find very fair,
The sum will be nothing, in truth, I declare.

113. A fisherman, being asked the depth of a lake, replied: "This pole when standing on the bottom reaches one foot out of the water, but if the top is moved through an arc of 30°, it becomes level with the surface of the water." How deep is the lake?

114. What is the shape of a square inch? Of an inch square?

115. What integer added to itself is greater than its square?

116. What number added to itself is equal to its square?

117. What number is it that can be multiplied by 1, 2, 3, 4, 5, or 6, and no new figures appear in the results?

118. $3+3-3+3\times 3-3\div 3\times 0=?$

119. Write any number of yards, feet, and inches. Reverse this and subtract from the original. Reverse the remainder and add to the remainder. The sum will in every case be 12 yards, 1 foot, 11 inches. The number of inches first written should not exceed the number of yards.

120. The Numbers 37 and 73

When the number 37 is multiplied by each of the figures of arithmetical progression, 3, 6, 9, 12, 15, 18, 21, 24, 27, all the products which result from it are composed of three repetitions of the same figure; and the sum of those figures is equal to that by which you multiplied the 37.

$$\begin{array}{r}37\\3\\\hline 111\end{array}\qquad\begin{array}{r}37\\6\\\hline 222\end{array}\qquad\begin{array}{r}37\\9\\\hline 333\end{array}\qquad\begin{array}{r}37\\12\\\hline 444\end{array}\qquad\begin{array}{r}37\\15\\\hline 555\end{array}$$

$$\begin{array}{r}37\\18\\\hline 666\end{array}\qquad\begin{array}{r}37\\21\\\hline 777\end{array}\qquad\begin{array}{r}37\\24\\\hline 888\end{array}\qquad\begin{array}{r}37\\27\\\hline 999\end{array}$$

If the number 73 be multiplied by each of the numbers of arithmetical progression, 3, 6, 9, 12, 15, 18, 21, 24, 27, the six products which result from this multiplication are terminated by one of the nine different figures, 1, 2, 3, 4, 5, 6, 7, 8, 9. These figures will be found in the reverse order to that of the progression.

121. Arrange the figures 1, 2, 3, 4, 5, 6, 7, 8, and 9 so their sum will be 100.

122. Arrange the first sixteen digits in a square so that they may count 34 in every straight line.

123. Arrange the figures 1 to 9, inclusive, in a triangle so as to count 20 in every straight line.

124. Arrange the figures 1 to 9, inclusive, in a circle, using one in the center, so as to count 15 in every straight line.

125. Arrange the figures 1 to 19, inclusive, in a circle, using one in the center, so as to count 30 in every straight line.

126. Arrange the figures 1 to 9, inclusive, in a triangle, so as to count 17 in every straight line.

127. Arrange the figures 1 to 9, inclusive, in a square so as to count 15 in every straight line.

128.

25	6	7	24	3
4	10	17	12	22
5	15	13	11	21
8	14	9	16	18
23	20	19	2	1

A Bordered Magic Square

129. "If you multiply the number of Jacob's sons by the number of times which the Israelites compassed Jericho, and add to the product the number of measures of barley which Boaz gave Ruth, divide this by the number of Haman's sons, subtract the number of each kind of clean beasts that went into the ark, multiply by the number of men that went to seek Elijah after he was taken to heaven; subtract from this Joseph's age at the time he stood before Pharaoh, add the number of stones in David's bag when he killed Goliath; subtract the number of furlongs that Bethany was distant from Jerusalem, divide by the number of anchors cast out when Paul was shipwrecked, subtract the number of persons saved in the ark, and the answer will be the number of pupils in my Sunday-school class." How many pupils are in the class?

130. MAGIC AGE TABLE

1	2	4	8	16	32
3	3	5	9	17	33
5	6	6	10	18	34
7	7	7	11	19	35
9	10	12	12	20	36
11	11	13	13	21	37
13	14	14	14	22	38
15	15	15	15	23	39
17	18	20	24	24	40
19	19	21	25	25	41
21	22	22	26	26	42
23	23	23	27	27	43
25	26	28	28	28	44
27	27	29	29	29	45
29	30	30	30	30	46
31	31	31	31	31	47
33	34	36	40	48	48
35	35	37	41	49	49
37	38	38	42	50	50
39	39	39	43	51	51
41	42	44	44	52	52
43	43	45	45	53	53
45	46	46	46	54	54
47	47	47	47	55	55
49	50	52	56	56	56
51	51	53	57	57	57
53	54	54	58	58	58
55	55	55	59	59	59
57	58	60	60	60	60
59	59	61	61	61	61
61	62	62	62	62	62
63	63	63	63	63	63

Key to Table. — Add together the figures at the top of each column in which the age is found, and the sum will be the age sought. *Example:* Hand the table to a lady and request her to tell you in which column or columns her age is found; if she says the first, fourth, and fifth, you can say it is 25 by mentally adding together the first figures of those three columns, and so on for any age up to 63.

131. How to Tell a Person's Age

Let the person whose age is to be discovered do the figuring. Suppose, for example, if it is a girl, that her age is 16, and that she was born in May. Let her put down the number of the month in which she was born and proceed as follows:

Number of month	5
Multiply by 2	10
Add 5	15
Multiply by 50	750
Then add her age, 16	766
Then subtract 365, leaving	401
Then add 115	516

She then announces the result, 516, whereupon she may be informed that her age is 16, and May, or the fifth month, is the month of her birth. The two figures to the right in the result will always indicate the age, and the remaining figure or figures the month in which her birthday comes.

132. A, B, and C were a mile at sea when a rifle was fired on shore. A heard the report, B saw the smoke, and C saw the bullet strike the water near them. Who first knew of the discharge of the rifle?

133. A Queer Trick of Figures

Put down the number of your living brothers.
Double the number.
Add 3.
Multiply the result by 5.
Add the number of your living sisters.
Multiply the result by 10.
Add the number of dead brothers and sisters.
Subtract 150 from the result.
The right-hand figure will be the number of deaths.
The middle figure will be the number of living sisters.
The left-hand figure will be the number of living brothers.

134. Find perfect square numbers, each containing all the 10 digits, under the following conditions:

(1) The least square possible.
(2) The greatest square containing no repeated digit.
(3) The least square which, when reversed, is still a square.
(4) The least square which is unaltered by reversal.

135. A house and a barn are 20 rods apart; the house is 10 rods and the barn 6 rods from a straight brook. What is the length of the shortest path by which one can go from the house to the brook and take water to the barn?

136. A and B dig a ditch for $10; A can dig as fast as B can shovel out the dirt, and B can dig twice as fast as A can shovel. How should they divide the $10?

137. Three Series of Remarkable Numbers

1×9 plus $1 = 10$
12×9 plus $2 = 110$
123×9 plus $3 = 1110$
1234×9 plus $4 = 11110$
12345×9 plus $5 = 111110$
123456×9 plus $6 = 1111110$
1234567×9 plus $7 = 11111110$
12345678×9 plus $8 = 111111110$
123456789×9 plus $9 = 1111111110$

1×9 plus $2 = 11$
12×9 plus $3 = 111$
123×9 plus $4 = 1111$
1234×9 plus $5 = 11111$
12345×9 plus $6 = 111111$
123456×9 plus $7 = 1111111$
1234567×9 plus $8 = 11111111$
12345678×9 plus $9 = 111111111$
123456789×9 plus $10 = 1111111111$

1 × 8 plus 1 = 9
12 × 8 plus 2 = 98
123 × 8 plus 3 = 987
1234 × 8 plus 4 = 9876
12345 × 8 plus 5 = 98765
123456 × 8 plus 6 = 987654
1234567 × 8 plus 7 = 9876543
12345678 × 8 plus 8 = 98765432
123456789 × 8 plus 9 = 987654321

138. At 10 A.M. a train leaves London for Edinburgh running at 50 miles an hour. At the same time another train leaves Edinburgh for London, traveling at 40 miles an hour. Which train is nearer London when they meet?

139. The asterisks in the incomplete sum printed below indicate missing figures. Find all the missing figures.

1*32271
52*4
63**74
88*47
305417
2*3547*
4,107,303

140. Determine the missing digits in the following sum in multiplication:

1*46
*5
6730
107*8
114,410

141. In a long division sum the dividend is 529,565, and the successive remainders from the first to the last are 246, 222, and 542. Find the divisor and the quotient.

142. The sum of two numbers consisting of the same three digits in reverse order is 1170, and their difference is divisible by 8. Find the numbers.

143. A girl was given a number to multiply by 409, but she placed the first figure of her product by 4 below the second figure from the right instead of below the third. Her answer was wrong by 328,320. Find the multiplicand.

144. I have a board $1\frac{1}{2}$ inches thick, whose surface contains $49\frac{5}{8}$ square feet. Find the edge of a cubical box made of it.

145. Write one billion by the Roman notation.

146. Each of two sons inherit 30 %, and each of two daughters 20 %, of a parallelogrammatic plantation, containing 100 acres, and having an open ditch on its long diagonal. The four divisions are to corner somewhere in the ditch, and each is to have a side of the plantation in its boundary. Locate this common corner.

147. Why is the difference between any common number of three digits and one containing the same digits in reversed order, always divisible by 9, 11, and the difference of the extreme digits?

148. Required with six 9's to express the number 100.

149. THE LUCKY NUMBER

Many persons have what they consider a "lucky" number. Show such a person the row of figures subjoined:

1, 2, 3, 4, 5, 6, 7, 9

(consisting of the numerals from 1 to 9 inclusive, with the 8 only omitted), and inquire what is his lucky or favorite number. He names any number he pleases from 1 to 9, say 7. You reply that, as he is fond of sevens, he shall have plenty of them, and accordingly proceed to multiply the series above given by such a number that the resulting product consists of sevens only.

Required to find (for each number that may be selected) the multiplier which will produce the above result.

150. Father and son are aged 71 and 34 respectively. At what age was the father three times the age of his son? and at what age will the latter have reached half his father's age?

151. There is a number consisting of two digits; the number itself is equal to five times the sum of its digits, and if 9 be added to the number, the position of its digits is reversed. What is the number?

152. THE EXPUNGED NUMERALS

Given the sum following:

$$\begin{array}{r} 111 \\ 333 \\ 555 \\ 777 \\ \underline{999} \end{array}$$

Required, to strike out nine of the above figures, so that the total of the remaining figures shall be 1111.

153. A Grayson County widower married a Denton County widow; each had children. Ten years later a domestic tornado prevailed in the back yard in which the present family of a dozen children were involved. Mother to father: "Your children and my children are picking at our children." If the parents now have each nine children of their own, how many came into the family in these ten years?

154. Some of the numbers differing from their logarithms only in the position of the decimal point.

$$\log 1.3712885742 = .13712885742$$
$$\log 237.5812087593 = 2.375812087593$$
$$\log 3550.2601815865 = 3.5502601815865$$

155. Consecutive numbers whose squares have the same digits:

$13^2 = 169$ $157^2 = 24649$ $913^2 = 833569$
$14^2 = 196$ $158^2 = 24964$ $914^2 = 835396$

156. To arrange the ten digits additively so as to make 100.

157. Express the numbers from 1 to 30 inclusive by using for each number four 4's.

158. Invert the figures of any three-place number; divide the difference between the original number and the inverted number by 9; and you may read the quotient forward or backward.

159. Write a number of three or more places, divide by 9, and tell me the remainder; erase one figure, not zero, divide the resulting number by 9, tell me the remainder, and I will tell you the figure erased.

160. Can a fraction whose numerator is less than its denominator be equal to a fraction whose numerator is greater than its denominator?

161. Show why 8 must be a factor of the product of any two consecutive even numbers.

162. A and B take a job of digging potatoes for $ 5. B can pick up as fast as A digs, but if B digs and A picks them, B must begin digging $\frac{1}{2}$ day before A begins picking, in order that each may complete his work at the same time. How shall they divide the money?

163. A and B are employed to dig a ditch 100 rods long for $ 200. A is to get $ 1.75 per rod and B $ 2.25 per rod. How much will each have to dig so as to be entitled to an equal share of the money?

164. If an egg balances with three quarters of an egg and three quarters of an ounce, find the weight of an egg.

165. A farmer had six pieces of chain of 5 links each, which he wanted made into an endless piece of 30 links. If it costs a cent to cut a link and costs a cent to weld it, what did it cost him?

166. A vessel of water full to the brim weighs 20 pounds. A 5-pound live fish is put into the vessel. Has the weight of the vessel of water been increased or diminished?

167. What is the most economical form of a tank designed to hold 1000 cubic inches?

168. "Johnnie, my boy," said a successful merchant to his little son, "it is not what we pay for things, but what we get for them that makes good business. I gained ten per cent on that fine suit of clothes, while if I had bought it ten per cent cheaper and sold it for twenty per cent profit, it would have brought a quarter of a dollar less money. Now, what did I get for that suit?"

— From "Our Puzzle Magazine."

169. While discussing practical ways and means with his good wife, Farmer Jones said: "Now, Maria, if we should sell off seventy-five chickens as I propose, our stock of feed would last just twenty days longer, while if we should buy a hundred extra fowl, as you suggest, we would run out of chicken feed fifteen days sooner." How many chickens had they?

— From "Our Puzzle Magazine."

170. Suppose that a bird weighing 1 ounce flies into a box with only one small opening, and without resting continues to fly round and round in the box; does it increase or lessen the weight of the box?

171. John can weed a row of potatoes while James digs three; but James can weed a row while John digs a row. If they get $10 for their work, how should it be divided between them?

172. THE WATCH TRICK

The following is a well-known way of indicating on a watch dial an hour selected by a person. The hour is tapped by a pencil beginning at VII and proceeding backwards round the dial to VI, V, IV, etc., and the person who selected the number counts the taps, reckoning from the hour selected. Thus, if he selected VIII, he would reckon the first tap as the 9th; then the 20th tap as reckoned by him will be on the hour chosen.

It is obvious that the first seven taps are immaterial, but the eighth tap must be on XII.

173. What is a third and a half of a third of 10?

174. (i) Write down a number thought of; (ii) add or subtract any number you wish; (iii) multiply, or divide by any number you wish; (iv) multiply by any multiple of 9; (v) cross out any digit except a naught; (vi) give me the sum of the remaining digits, and I will give you the figure struck out.

175. A banker going home to dinner saw a $10 bill on the curbstone. He picked it up, noted the number, and went home to dinner. While at home his wife said that the butcher had sent a bill amounting to $10. The only money he had was the bill he had found, which he gave to her, and she paid the butcher. The butcher paid it to a farmer for a calf, the farmer paid it to the merchant, who in turn paid it to a washerwoman, and she, owing the bank a note of $10 went to the bank and paid the note. The banker recognized the bill as the one he had found, and which to that time had paid $50 worth of debt. On careful examination he discovered that the bill was counterfeit. Now what was lost in the transaction, and by whom?

176. What is the difference between a mile square and a square mile?

177. A Multiplication Trick

Here is a little trick in multiplication that may amuse you. Ask a friend to write down the numbers 12345679, omitting the number 8. Then tell him to select any one figure from the list, multiply it by 9, and with the answer to this sum multiply the whole list — thus assuming that he selects either the figure 4 or 6.

Select 4 × 9 = 36.	Select 6 × 9 = 54.
12345679	12345679
36	54
74074074	49382716
37037037	61728395
444444444	666666666

You see the answer of the sum is composed of figures similar to the one selected.

178. Cook was within 10 miles of the north pole and Peary was also within 10 miles of the pole, but 20 miles from Cook. What direction was Peary from Cook? Suppose Peary threw a ball at Cook and hit him. In what direction did the ball go?

179. A man has 12 pieces of chain of 3 links each. He takes them to a blacksmith to unite them into one circular or endless chain. If it costs 2 cents to cut a link and 2 cents to weld a link, what should the blacksmith charge for the job?

180. Take 2 pennies, face upwards on a table and edges in contact. Suppose that one is fixed and that the other rolls on it without slipping, making one complete revolution round it and returning to its initial position. How many revolutions round its own center has the rolling coin made?

181.

From six you take nine;
And from nine you take ten;
Then from forty take fifty,
And six will remain.

182. A room is 30 feet long, 12 feet wide, and 12 feet high. At one end of the room, 3 feet from the floor, and midway from the sides, is a spider. At the other end, 9 feet from the floor, and midway from the sides, is a fly. Determine the shortest path the spider can take to capture the fly by crawling.

183. A GEOMETRICAL PARADOX

A stick is broken at random into 3 pieces. It is possible to put them together into the shape of a triangle provided the length of the longest piece is less than the sum of the other 2 pieces; that is, provided the length of the longest piece is less than half the length of the stick. But the probability that a fragment of a stick shall be half the original length of the stick is $\frac{1}{2}$. Hence the probability that a triangle can be constructed out of the 3 pieces into which the stick is broken is $\frac{1}{2}$.

184. A GEOMETRICAL FALLACY

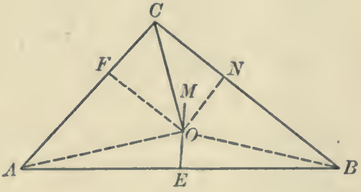
Proposition. — All triangles are isosceles.

Given, any triangle ABC.

To prove triangle ABC is isosceles.

Proof. — Draw ME perpendicular to AB at the mid-point of AB; and draw CO, the bisector of the angle C, intersecting the line ME in O.

Draw the perpendiculars, OF and ON, to the sides AC and BC, respectively.



Then $ON = OF$.

$\therefore CF = CN$.

Join A and O; also join O and B.

Then $AO = BO$.

$\therefore$ the triangles AOF and OBN are congruent.

(Being right triangles having $AO = BO$ and $OF = ON$.)

$\therefore AF = BN$.

$\therefore AF + FC = CN + NB$, or $AC = BC$.

185. Three men robbed a gentleman of a vase containing 24 ounces of balsam. While running away they met in a forest with a glass seller, of whom in a great hurry they purchased three vessels. On reaching a place of safety they wished to divide the booty, but they found that their vessels contained 5, 11, and 13 ounces respectively. How could they divide the balsam into equal portions?

186. A man bets $\frac{1}{m}$th of his money on an even chance (say tossing heads or tails with a coin); he repeats this again and again, each time betting $\frac{1}{m}$th of all the money then in his possession. If, finally, the number of times he has won is equal to the number of times he has lost, has he gained or lost by the transaction?

187. What like fractions of a pound, of a shilling, and of a penny, when added together, make exactly a pound?

188. Required to subtract 45 from 45 in such a manner that there shall be a remainder of 45.

189. Any prime number, which, divided by 4, leaves a remainder 1 is the sum of two perfect squares.

Below is given a list of all prime numbers below 400 which, being divided by 4, leave a remainder of 1:

$5 = 4 + 1 = 2^2 + 1^2$
$13 = 9 + 4 = 3^2 + 2^2$
$17 = 16 + 1 = 4^2 + 1^2$
$29 = 25 + 4 = 5^2 + 2^2$
$37 = 36 + 1 = 6^2 + 1^2$
$41 = 25 + 16 = 5^2 + 4^2$
$53 = 49 + 4 = 7^2 + 2^2$
$61 = 36 + 25 = 6^2 + 5^2$
$73 = 64 + 9 = 8^2 + 3^2$
$89 = 64 + 25 = 8^2 + 5^2$
$97 = 81 + 16 = 9^2 + 4^2$
$101 = 100 + 1 = 10^2 + 1^2.$
$109 = 100 + 9 = 10^2 + 3^2$
$113 = 64 + 49 = 8^2 + 7^2$
$137 = 121 + 16 = 11^2 + 4^2$
$149 = 100 + 49 = 10^2 + 7^2$
$157 = 121 + 36 = 11^2 + 6^2$
$173 = 169 + 4 = 13^2 + 2^2$
$181 = 100 + 81 = 10^2 + 9^2$
$193 = 144 + 49 = 12^2 + 7^2$

$197 = 196 + 1 = 14^2 + 1^2$
$229 = 225 + 4 = 15^2 + 2^2$
$233 = 169 + 64 = 13^2 + 8^2$
$241 = 225 + 16 = 15^2 + 4^2$
$257 = 256 + 1 = 16^2 + 1^2$
$269 = 169 + 100 = 13^2 + 10^2$
$277 = 196 + 81 = 14^2 + 9^2$
$281 = 256 + 25 = 16^2 + 5^2$
$293 = 289 + 4 = 17^2 + 2^2$
$313 = 169 + 144 = 13^2 + 12^2$
$317 = 196 + 121 = 14^2 + 11^2$
$337 = 256 + 81 = 16^2 + 9^2$
$349 = 324 + 25 = 18^2 + 5^2$
$353 = 289 + 64 = 17^2 + 8^2$
$373 = 324 + 49 = 18^2 + 7^2$
$389 = 289 + 100 = 17^2 + 10^2$
$397 = 361 + 36 = 19^2 + 6^2$

190. Any number, less the sum of its digits, is divisible by 9.

Proof. Let a represent the units, b the tens, c the hundreds, d the thousands, and so on.

Then,
$$\begin{array}{lcrcl} a \text{ units} & = & a \text{ units} & = & 0 + a \text{ units} \\ b \text{ tens} & = & 10\, b \text{ units} & = & 9\, b + b \text{ units} \\ c \text{ hundreds} & = & 100\, c \text{ units} & = & 99\, c + c \text{ units} \\ d \text{ thousands} & = & 1000\, d \text{ units} & = & 999\, d + d \text{ units} \\ \hline \multicolumn{5}{l}{\text{The number } = 999\, d + 99\, c + 9\, b + a + b + c + d \text{ units}} \end{array}$$

The sum of the digits $= a + b + c + d$ units. Subtracting, we have a remainder of $999\, d + 99\, c + 9\, b$.

Since $999\, d + 99\, c + 9\, b$ is a multiple of 9, it is divisible by 9.

191. Two persons were born Jan. 1, 1830, and both died Jan. 1, 1885; yet one lived 10 days longer than the other. Explain how this could be possible.

192. Two men are 20 miles apart. They walk in the same direction, at the same rate of speed, for the same length of time; they are then 30 miles apart. Show three ways in which this could be possible.

193. Two men start from the same place at the same time and go in the same direction for the same length of time at the same rate of speed. When they have gone $\frac{1}{2}$ the journey they find they are about 8000 miles apart, yet they complete their journeys at the same time. How is this possible?

194. Every direction is south except up and down. Where am I?

195. A boy plants a grain of corn 5 inches under the soil. The first night it sprouts and grows $\frac{1}{2}$ the distance, and continues to grow $\frac{1}{2}$ the remaining distance each night following. How long before it will come up?

196. Sterling Jones, a heavy boy, weighs 20 pounds plus $\frac{1}{4}$ of his own weight, plus $\frac{1}{8}$ of his own weight, plus $\frac{1}{16}$ of his own weight . . . to infinity. What is his weight?

197. Express the number 10 by using five 9's in 4 different ways.

198. THE PARADOX OF TRISTRAM SHANDY

Tristram Shandy took 2 years writing the history of the first 2 days of his life, and lamented that, at this rate, material would accumulate faster than he could deal with it, so that he could never come to an end, however long he lived. But had he lived long enough, and not wearied of his task, then, even if his life had continued as eventfully as it began, no part of his biography would remain unwritten. For if he wrote the events of the first day in the first year, he would write the events of the *n*th day in the *n*th year, hence in time the events of any assigned day would be written, and therefore no part of his biography would remain unwritten.

— From Ball's "Mathematical Recreations and Essays."

199. SWIFT'S BIOLOGICAL DIFFICULTY

Great fleas have little fleas upon their backs to bite 'em,
And little fleas have lesser fleas, and so ad infinitum.
And the great fleas themselves, in turn, have greater fleas to go on;
While these have greater still, and greater still, and so on.

— DE MORGAN.

200. A couple of dice are thrown. The thrower is invited to double the points of one of the dice (whichever he pleases), add 5 to the result, multiply by 5, and add the points of the second die. He states the total, when any one knowing the secret can instantly name the points of the two dice. How is it done?

201. Three dice are thrown. The thrower is asked to multiply the points of the first die by 2, add 5 to the result, multiply by 5, add the points of the second die, multiply the total by 10, and add the points of the third die. He states the total. Name the points of the three dice.

202. A man has 21 casks. Seven are full of wine; 7 half full, and 7 empty. How can he divide them, without transferring any portion of the liquid from cask to cask, among his three sons, — Sam, John, and James, — so that each shall have an equal quantity of wine and also an equal number of casks?

203. Three beautiful ladies have for husbands three men, who are as jealous as they are young, handsome, and gallant. The party are traveling, and find on the bank of a river, over which they have to pass, a small boat which can hold no more than two persons. How can they cross, it being agreed that no woman shall be left in the society of a man unless her husband is present?

204. A certain number is divisible into four parts, in such manner that the first is 500 times, the second 400 times, and the third 40 times as much as the last and smallest part. What is the number and what are the several parts?

205. What is the smallest number which, divided by 2, will give a remainder of 1; divided by 3, a remainder of 2; divided by 4, a remainder of 3; divided by 5, a remainder of 4; divided by 6, a remainder of 5; divided by 7, a remainder of 6; divided by 8, a remainder of 7; divided by 9, a remainder of 8; and divided by 10, a remainder of 9?

206. Given, five squares of paper or cardboard, alike in size. Required, so to cut them that by rearrangement of the pieces you can form one large square.

207. Given a board 3 feet long and 1 foot wide. Required to cover a hole 2 feet by 1 foot 6 inches, by not cutting the board into more than two pieces.

208. Given a board 15 inches long and 3 inches wide. How is it possible to cut it so that the pieces when rearranged shall form a perfect square?

209. Place the numbers 1 to 19 inclusive on the sides of the six equilateral triangles which form a regular hexagon, so that the sum on every side will be the same.

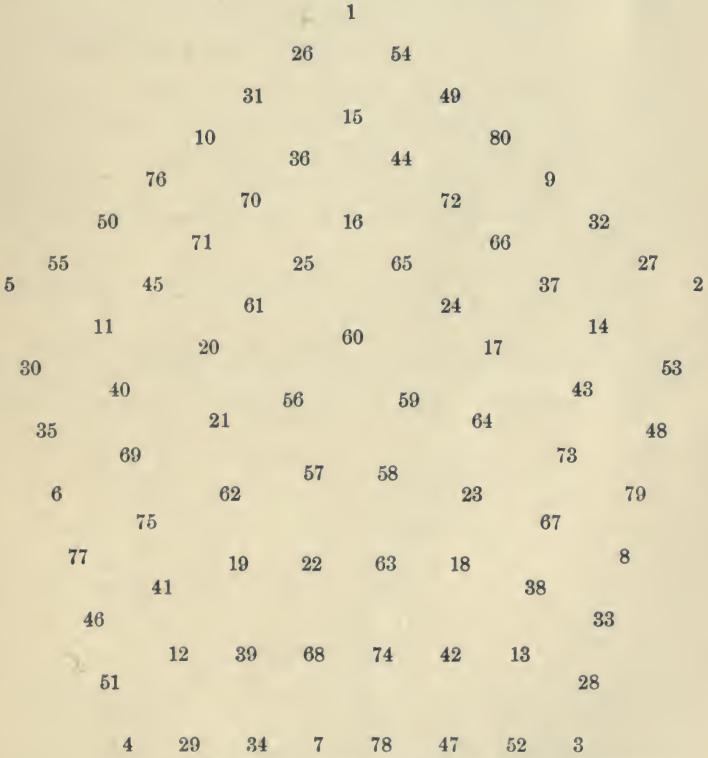
210. 15 Christians and 15 Turks, being at sea in one and the same ship in a terrible storm, and the pilot declaring a necessity of casting one half of those persons into the sea, that the rest might be saved; they all agreed that the persons to be cast away should be set out by lot after this manner, viz., the 30 persons should be placed in a round form like a ring, and then beginning to count at one of the passengers, and proceeding circularly, every ninth person should be cast into the sea, until of the 30 persons there remained only 15. The question is, how those 30 persons should be placed, that the lot might infallibly fall upon the 15 Turks and not upon any of the 15 Christians.

211. SOME VERY OLD PROBLEMS

Heap, its seventh, its whole, it makes 19.

— From Ahmes, Collection of Problems, made in Egypt between 3400 B.C. and 1700 B.C.

212. The numbers from 1 to 80 admit of being formed about a point as common center into four pentagons, such that each side of the first pentagon from within contains two numbers, each side of the second pentagon four numbers, each of the third six numbers, and each side of the fourth, outermost pentagon eight numbers. The sum of the numbers of each side of the second pentagon is 122, the sum of those of each side of the third pentagon is 248, and that of those of each side of the fourth pentagon 254. Furthermore, the sum of any four corner numbers lying in the same straight line with the center, is also the same; namely, 92.


1
26 54
31 49
15
10 80
36 44
76 9
70 72
50 16 32
71 66
55 25 65 27
5 45 37 2
61 24
11 14
60
20 17
30 53
40 43
56 59
35 21 64 48
69 73
57 58
6 62 23 79
75 67
77 8
19 22 63 18
41 38
46 33
12 39 68 74 42 13
51 28
4 29 34 7 78 47 52 3

— From “Essays and Recreations” by Schubert.

213. A mule and a donkey were walking along, laden with corn. The mule says to the donkey, "If you gave me one measure, I should carry twice as much as you. If I gave you one, we should both carry equal burdens." Tell me their burdens, O most learned master of geometry.

— A riddle attributed to Euclid. From "Palatine Anthology," 300 A.D.

214. What part of the day has disappeared if the time left is twice two thirds of the time passed away?

— "Palatine Anthology," 300 A.D.

215. The square root of half the number of bees in a swarm has flown out upon a jessamine bush, $\frac{8}{9}$ of the whole swarm has remained behind; one female bee flies about a male that is buzzing within a lotus flower into which he was allured in the night by its sweet odor, but is now imprisoned in it. Tell me the number of bees.

— From "Lilavati," a Chapter in Bhaskara's great work, written in 1150 A.D.

216. Find the keyword in the following problem in "Letter Division."

$$
\begin{array}{l}
\underline{\texttt{CPN)}}\texttt{AOUIERT}\underline{\texttt{(PCAAU}}\\
\phantom{\texttt{CPN)}}\underline{\texttt{CPN}}\\
\phantom{\texttt{CPN)}}\texttt{PIUI}\\
\phantom{\texttt{CPN)}}\underline{\texttt{PUCN}}\\
\phantom{\texttt{CPN)A}}\texttt{RRIE}\\
\phantom{\texttt{CPN)A}}\underline{\texttt{RNAN}}\\
\phantom{\texttt{CPN)AO}}\texttt{REER}\\
\phantom{\texttt{CPN)AO}}\underline{\texttt{RNAN}}\\
\phantom{\texttt{CPN)AOU}}\texttt{RIRT}\\
\phantom{\texttt{CPN)AOU}}\underline{\texttt{RCUN}}\\
\phantom{\texttt{CPN)AOUI}}\texttt{EUT}
\end{array}
$$

Note. — For other problems of this kind, see "Div-A-Let," by W. H. Vail, Newark, N. J.

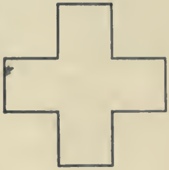
217. Demochares has lived a fourth of his life as a boy; a fifth as a youth; a third as a man; and has spent 13 years in his dotage. How old is he?

— From a collection of questions by Metrodorus, 310 A.D.

218. Beautiful maiden with beaming eyes, tell me, as thou understandest the right method of inversion, which is the number which multiplied by 3, then increased by $\frac{3}{4}$ of the product, divided by 7, diminished by $\frac{1}{3}$ of the quotient, multiplied by itself, diminished by 52, the square root extracted, addition of 8, and division by 10, gives the number 2?

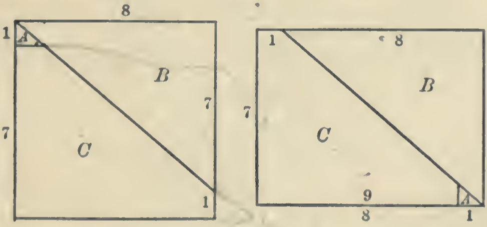
— From "Lilavati."

219. Given a piece of cardboard in the form of a Greek or equal-armed cross, as shown in the figure. Required, by two straight cuts, so to divide it that the pieces when reunited shall form a square.



220. To show geometrically that $1 = 0$.

First Solution. Take a square that is 8 units on a side, and cut it into three parts, *A*, *B*, and *C*, as shown in the left-hand figure. Fit these parts together as in the right-hand figure.



Now the square is 8 units on a side, and therefore contains 64 small squares, while the rectangle is 9 units long and 7 units wide, and therefore contains 63 small squares.

Each of the figures are made up of *A*, *B*, and *C*.

In the square $A + B + C = 64.$
In the rectangle $A + B + C = 63.$

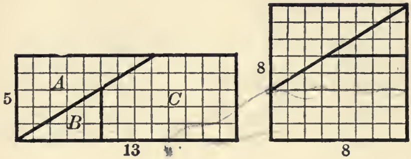
$$\therefore 64 = 63.$$

(Things equal to the same thing are equal to each other.)

$$\therefore 1 = 0.$$

(By subtracting 63 from each side of the equation.)

Second Solution. Take a square that is 8 units on a side, and cut it into three parts, A, B, and C, as shown in the right-hand figure. Fit these parts together as in the left-hand figure.



Now the square is 8 units on a side, and therefore contains 64 small squares, while the rectangle is 13 units long and 5 units wide, and therefore contains 65 small squares.

Each of the figures are made up of A, B, and C.

In the rectangle $A + B + C = 65.$
In the square $A + B + C = 64.$

$$\therefore 65 = 64.$$

$$\therefore 1 = 0.$$

221. To prove that $1 = 200$.

Let $a = b = 10.$
Then $a^2 - b^2 = 0,$
and $a^4 - b^4 = 0.$

$$\therefore a^2 - b^2 = a^4 - b^4.$$

$$\therefore a^2 - b^2 = (a^2 - b^2)(a^2 + b^2).$$

$$\therefore 1 = a^2 + b^2.$$

$$\therefore 1 = 10^2 + 10^2.$$

$$\therefore 1 = 200.$$

NOTE. — If $a = 1$, $1 = 2$; if $a = 2$, $1 = 8$; if $a = 3$, $1 = 18$; etc.

* **222.** To prove that $1 = 2000$.

Let $a = b = 10.$

Then $a^3 - b^3 = 0,$

and $a^6 - b^6 = 0.$

$$\therefore\ a^3 - b^3 = a^6 - b^6.$$

(Things equal to the same things are equal to each other.)

$$\therefore\ 1 = a^3 + b^3.$$

(Dividing by $a^3 - b^3$.)

$$\therefore\ 1 = 10^3 + 10^3.$$

$$\therefore\ 1 = 2000.$$

Note. — If $a = 1$, $1 = 2$; if $a = 2$, $1 = 16$; if $a = 3$, $1 = 54$; etc. Also many other problems may be made similar to problems Nos. 221 and 222.

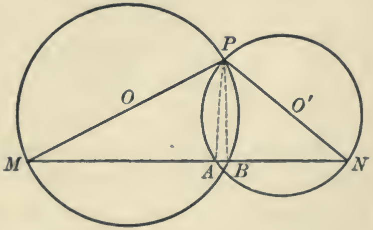
* 223. ANOTHER GEOMETRICAL FALLACY

To prove that it is possible to let fall two perpendiculars to a line from an external point.

Take two intersecting circles with centers O and O'. Let one point of intersection be P, and draw the diameters PM and PN.

Draw MN cutting the circumferences at A and B. Then draw PA and PB.

Since $\angle PBM$ is inscribed in a semicircle, it is a right angle. Also since $\angle PAN$ is inscribed in a semicircle, it is a right angle.



$\therefore$ PA and PB are both $\perp$ to MN.

224. Given three or more integers, as 30, 24, and 16; required to find their greatest integral divisor that will leave the same remainder.

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* The exposing of fallacies has been left to the student. They should be studied in every High School and College.

225. To Prove that You are as Old as Methuselah

Proof:

Let $x =$ Methuselah's age.

Let $y =$ your age.

Let $s =$ the sum.

Then $x + y = s.$

$$\therefore (x+y)(x-y) = s(x-y).$$

$$\therefore x^2 - y^2 = sx - sy.$$

$$\therefore x^2 - sx = y^2 - sy.$$

$$\therefore x^2 - sx + \frac{s^2}{4} = y^2 - sy + \frac{s^2}{4}.$$

$$\therefore \left(x - \frac{s}{2}\right)^2 = \left(y - \frac{s}{2}\right)^2.$$

$$\therefore x - \frac{s}{2} = y - \frac{s}{2}.$$

$$\therefore x = y.$$

226. How many shoes would it take for the people of a town if one-third of them had but one foot and one half the remainder went barefoot?

227. The Spider and the Four Gnats

On a suspended piece of glass 10 inches long, 4 inches wide, and 4 inches high is a spider and four gnats. The spider is on one end $\frac{1}{3}$ inch from the bottom and midway between the sides. The gnats are on the other end. Three of them are $\frac{1}{3}$ inch, $\frac{2}{3}$ inch, and 1 inch, respectively, from the top and midway between the sides. The fourth is $1\frac{30}{37}$ inches from the top and on an edge.

Determine the shortest path possible, by way of the six faces of the piece of glass, for the spider to catch the four gnats and return to the place from which he started.

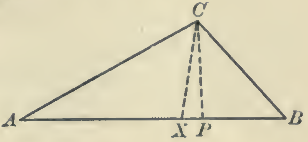
228. What difference would there be in the weight of a perfectly air-tight bird cage, depending on whether the bird were sitting on the perch or flying about?

229. To prove that part of a line equals the whole line.

Take a triangle ABC, and draw $CP \perp$ to AB.

From C draw CX, making $\angle ACX = \angle B$.

Then $\triangle$ ABC and ACX are similar.



$$\therefore \triangle ABC : \triangle ACX = \overline{BC}^2 : \overline{CX}^2.$$

Furthermore, $\triangle ABC : \triangle ACX = AB : AX$.

$$\therefore \overline{BC}^2 : \overline{CX}^2 = AB : AX,$$

or
$$\overline{BC}^2 : AB = \overline{CX}^2 : AX.$$

But
$$\overline{BC}^2 = \overline{AC}^2 + \overline{AB}^2 - 2\,AB \cdot AP,$$

and
$$\overline{CX}^2 = \overline{AC}^2 + \overline{AX}^2 - 2\,AX \cdot AP.$$

$$\therefore \frac{\overline{AC}^2 + \overline{AB}^2 - 2\,AB \cdot AP}{AB} = \frac{\overline{AC}^2 + \overline{AX}^2 - 2\,AX \cdot AP}{AX},$$

or
$$\frac{\overline{AC}^2}{AB} + AB - 2\,AP = \frac{\overline{AC}^2}{AX} + AX - 2\,AP.$$

$$\therefore \frac{\overline{AC}^2}{AB} - AX = \frac{\overline{AC}^2}{AX} - AB,$$

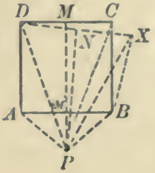
or
$$\frac{\overline{AC}^2 - AB \cdot AX}{AB} = \frac{\overline{AC}^2 - AB \cdot AX}{AX}.$$

$$\therefore AB = AX.$$

— From Wentworth and Smith's "Geometry."

230. To prove that part of an angle equals the whole angle.

Take a square $ABCD$, and draw $MM'P$, the $\perp$ bisector of CD. Then $MM'P$ is also the $\perp$ bisector of AB.



From B draw any line BX equal to AB.

Draw DX and bisect it by the $\perp$ NP. Since DX intersects CD, $\perp$s to these lines cannot be parallel, and must meet as at P.

Draw PA, PD, PC, PX, and PB.

Since MP is the $\perp$ bisector of CD, $PD = PC$.

Similarly, $PA = PB$, and $PD = PX$.

$\therefore PX = PD = PC$.

But $BX = BC$ by construction, and PB is common to $\triangle$s PBX and PBC.

$\therefore \triangle PBX$ is congruent to $\triangle PBC$, and $\angle XBP = \angle CBP$.

$\therefore$ the whole $\angle XBP$ equals the part, $\angle CBP$.

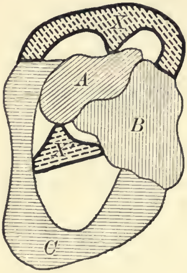
— From Wentworth and Smith's "Geometry."

231. The Four-color Map Problem

Not more than four colors are necessary in order to color a map of a country, divided into districts, in such a way that no two contiguous districts shall be of the same color.

Probably the following argument, though not a formal demonstration, will satisfy the reader that the result is true.

Let A, B, C be three contiguous districts, and let X be any other district contiguous with all of them. Then X must lie either wholly outside the external boundary of the area ABC or wholly inside the internal boundary; that is, it must occupy a position either like X or like X'. In either case every remaining occupied area in the figure is inclosed by the boundaries of not more than three districts; hence there is no possible way of drawing another area Y which shall be contiguous with A, B, C, and X. In other words, it is possible to draw on a plane four areas which are contiguous, but it is not possible to draw five such areas.



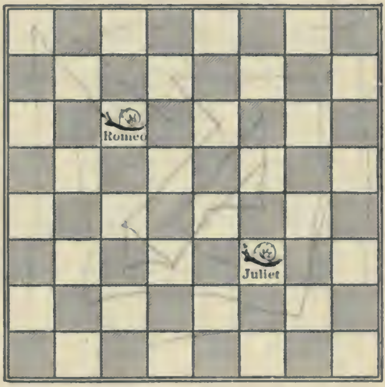
If A, B, C are not contiguous, each with the other, or if X is not contiguous with A, B, and C, it is not necessary to color them all differently, and thus the most unfavorable case is that already treated. Moreover, any of the above areas may diminish to a point and finally disappear without affecting the argument.

That we may require at least four colors is obvious from the above diagram, since in that case the areas A, B, C, and X would have to be colored differently.

A proof of the proposition involves difficulties of a high order, which as yet have baffled all attempts to surmount them. — From Ball's "Mathematical Recreations."

232. ROMEO AND JULIET

On a checker board are located two snails. They are Romeo and Juliet. Juliet is on her balcony waiting the arrival of her lover, but Romeo has been dining and forgets, for the life of him, the number of her house. The squares represent sixty-four houses, and the amorous swain visits every house once and only once before reaching his beloved.



Now make him do this with the fewest possible turnings. The snail can move up, down, and across the board and through the diagonals. Mark his track.

— From "Canterbury Puzzles."

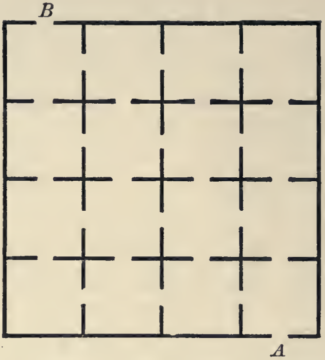
233. Find the exact dimensions of two cubes the sum of whose volumes will be exactly 17 cubic inches. Of course the cubes may be of different sizes.

234. I have two balls whose circumferences are respectively 1 foot and 2 feet. Find the circumferences of two other balls different in size whose combined volumes will exactly equal the combined volumes of the given balls.

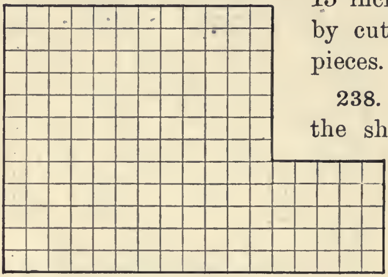
235. Can the number 11,111,111,111,111,111 be divided by any other integer except itself and unity?

236. My friend owns a house containing 16 rooms as indicated in the diagram.

While visiting him one day, he said to me, "Can you enter at the door A and pass out at the door B and enter every one of the 16 rooms once and only once?" Show how I might have done this.



237. Given a plank containing 169 square inches as shown below. Show how a hole 13 inches square may be covered by cutting the plank into three pieces.



238. Given a piece of cloth in the shape of an equilateral triangle. Required to cut it into four pieces that may be put together and form a perfect square.

239. A Short Method of Multiplication

Example. — Multiply 41,096 by 83.

The answer is found to be 3,410,968 by inspection. It will be observed that the answer is found by placing the last figure of the multiplier before the number and the first after it. Also if we prefix to 41,096 the number 41,095,890, repeated any number of times, the result may always be multiplied by 83 in this peculiar manner.

8 multiplied by 86 = 688.

Also to multiply 1,639,344,262,295,081,967,213,114,754,098,-360,655,737,704,918,032,787 by 71, all you have to do is to place another 1 at the beginning and another 7 at the end.

***240.**

THE SQUARE FALLACY

To prove that the diagonal of any square field equals the sum of any two sides.

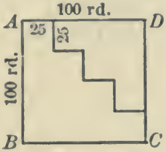


FIG. 1.

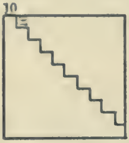


FIG. 2.

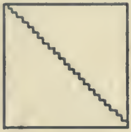


FIG. 3.

Given the square field *ABCD* with a side equal to 100 rods. The distance from *A* to *C* along two sides is 200 rods.

Now in Fig. 1 the distance from *A* to *C* along the diagonal path is 200 rods. In Fig. 2 the steps are smaller, yet the diagonal path is 200 rods long. In Fig. 3 the steps are very small, yet the distance must be 200 rods and would yet be if we needed a microscope to detect the steps. In this way we may go on straightening out the zigzag path until we ultimately reach a perfect straight line, and it therefore follows that the diagonal of a square equals the sum of any two sides. Can you expose the fallacy?

241. Given a rectangular block of wood 8 inches by 4 inches by 3¾ inches. Required to cut it into similar blocks 2½ inches by 1½ inches by 1¼ inches with the least possible waste. How many blocks can be had?

A TIME PROBLEM

242. A man who carries a watch in which the hour, minute, and second hands turn upon the same center was asked the time of day. He replied, "The three hands are at equal distances from one another and the hour hand is exactly 20-minute spaces ahead of the minute hand." Can you tell the time?

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* See footnote, page 95.

The Tree Planter

243. Are you a practical tree planter? If so, you are requested, (*a*) to show how sixteen trees may be planted in twelve straight rows, with four trees in every row, (*b*) to show how sixteen trees may be planted in fifteen straight rows, with four trees in every row.

244. Five persons can be seated in six different ways around a table in such a manner that any one person is seated only once between the same two persons. Show the manner of seating.

245. Seven persons may be seated in fifteen different ways around a table in such a manner that any one person is seated only once between the same two persons. Show the ways in which they might be seated.

246. On his morning stroll, Mr. Busybody encountered a laborer digging a hole. "How deep is that hole?" he asked. "Guess," replied the workingman, who stood in the hole. "My height is exactly five feet and ten inches."

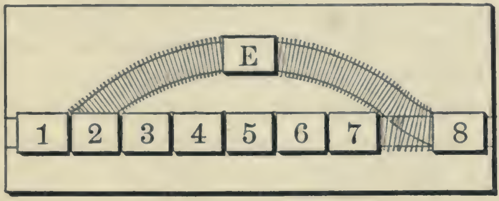
"How much deeper are you going?"

"I am going twice as deep," rejoined the laborer, "and then my head will be twice as far below ground as it now is above ground."

Mr. Busybody wants to know how deep that hole will be when finished.

247. One night three men, A, B, and C, stole a bag of apples and hid them in a barn over night, intending to meet in the morning to divide them equally. Some time before morning A went to the barn, divided the apples into three equal shares and had one apple too many, which he threw away. A took one share and put the others back into the bag. Soon after B came and did exactly as A had done. Then came C, who repeated what A and B had done before him. In the morning the three met, saying nothing of what they had done during the night. The remaining apples were divided into three equal shares, with still one apple too many. How many apples were there in the bag at the beginning?

248. The following diagram represents a section of a railway track with a siding. Eight cars are standing on the main



line in the order 1, 2, 3, 4, 5, 6, 7, 8, and an engine is standing on the side track. The siding will hold five cars, or four cars and the engine. The main line will hold only the eight cars and the engine. Also when all the cars and the engine are on the main line, only the one occupying the place of 8 can be moved on the siding. With 8 at the extremity, as shown, there is just room to pass 7 on the siding. The cars can be moved without the aid of the engine.

You are required to reverse the order of the cars on the main line so that they will be numbered 8, 7, 6, 5, 4, 3, 2, 1; and to do this by means which will involve as few transferences of the engine, or a car to or from the siding as are possible.

249. THE MYSTERIOUS ADDITION

To express the sum of five numbers, having given only the first.

Have a person write a number, say 55,369. Subtract two from the number, and place it before the remainder, giving 255,367, which is the sum of the numbers to be added. Each number is to contain the same number of figures as the first.

After the first number is expressed have the person write the second, say 38,465. Then write the third yourself, using such figures in the number, that if added to the figures in the number above will make nine. Have the person write the fourth number. Then write the fifth yourself in the same way as the third. These numbers added will give the required sum.

55,369
38,465
61,534
23,461
76,538
255,367

250. At the close of four and a half months' hard work, the ladies of a certain Dorcas Society were so delighted with the completion of a beautiful silk patchwork quilt for the dear curate that everybody kissed everybody else, except, of course, the bashful young man himself, who kissed only his sisters, whom he had called for, to escort home. There were just a gross of osculations altogether. How much longer would the ladies have taken over their needlework task if the sisters of the curate referred to had played lawn tennis instead of attending the meetings? Of course we must assume that the ladies attended regularly, and I am sure that they all worked equally well. A mutual kiss counts two osculations.

— From "Canterbury Puzzles."

251. The Arithmetical Triangle

This name has been given to a contrivance said to have originated or to have been perfected by the famous Pascal.

1							
2	1						
3	3	1					
4	6	4	1				
5	10	10	5	1			
6	15	20	15	6	1		
7	21	35	35	21	7	1	
8	28	56	70	56	28	8	1
etc.					etc.		

This peculiar series of numbers is thus formed: Write down the numbers 1, 2, 3, etc., as far as you please, in a vertical row. On the right hand of 2 place 1, add them together, and place 3 under the 1; then 3 added to 3 = 6, which place under the 3; 4 and 6 are 10, which place under the 6, and so on, as far as you wish. This is the second vertical row, and the third is formed from the second in a similar way.

This triangle has the property of informing us, without the trouble of calculation, how many combinations can be made, taking any number at a time, out of a larger number.

Suppose the question were that just given; how many selections can be made of 3 at a time, out of 8?

On the horizontal row commencing with 8, look for the third number; this is 56, which is the answer.

252. Twelve nests are in a circle. In each nest is only one egg. Required to begin at any nest, always going in the same direction, and pick up an egg, pass it over two other eggs, and place it in the next nest. This process is to be continued until six eggs have been removed and then six of the nests should contain two eggs each, and the other six should be empty. Show how this can be done by making the fewest possible revolutions around the nests.

253. A man in a city skyscraper, in a time of fire, made his escape by descending on a rope. He was 300 feet above the ground and had a rope only 150 feet long and $1\frac{1}{2}$ inches in diameter. Show how he made his escape without jumping from the window or dropping from the end of the rope.

254. A German farmer while visiting town bought a cask of wine containing 100 pints of pure wine. After reaching home he hid the cask in his barn thinking no one would find it. While away from home his neighbor found the cask and drew out 30 pints. Each time he drew out a pint he replaced it with a pint of pure water before drawing the next pint. How much wine was stolen?

255. While out fishing on a lake in a small boat I found myself without oars. I was two miles from shore. I had nothing to use to row the boat. Besides this there was no current to help me, for the water was perfectly smooth. I had nothing in the boat but a heavy trot-line one inch in diameter and six large fish. I could not swim and had no way of securing assistance. Was it possible for me to reach the shore under such circumstances? If so, how?

256. C's age at A's birth was $5\frac{1}{2}$ times B's age and now is equal to the sum of A's age and B's age. If A were 3 years younger or B 4 years older, A's age would be $\frac{3}{4}$ of B's age. Find the ages of A, B, and C. (Solve by arithmetic.)

257. What is the smallest sum of money in pounds, shillings, pence, and farthings that can be expressed by using each of the nine digits, 1, 2, 3, 4, 5, 6, 7, 8, and 9, once and once only?

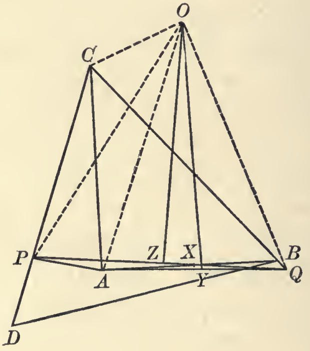
29	17	61	72
71	62	19	27
12	21	77	69
67	79	22	11

258. A REVERSIBLE MAGIC SQUARE.

The digits 0, 1, 2, 6, and 8, when turned upside down, can be read, 0, 1, 7, 9, and 8. It will be observed that this square when turned upside down is still magic.

259. To prove that part of an angle equals the whole angle.

Take a right triangle *ABC* and construct upon the hypotenuse *BC* an equilateral triangle *BCD*, as shown.



On *CD* lay off *CP* equal to *CA*.

Through *X*, the mid-point of *AB*, draw *PX* to meet *CB* produced at *Q*. Draw *QA*.

Draw the ⊥ bisectors of *QA* and *QP*, as *YO* and *ZO*. These must meet at some point O because they are $\perp$ to two intersecting lines.

Draw OQ, OA, OP, and OC.

Since O is on the $\perp$ bisector of QA, $\therefore OQ = OA$.

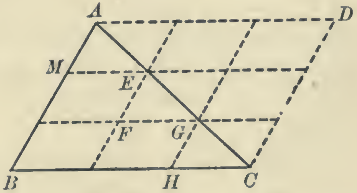
Similarly $OQ = OP$, and $\therefore OA = OP$.

But $CA = CP$, by construction, and $CO = CO$.

$\therefore \triangle AOC$ is congruent to $\triangle POC$, and $\angle ACO = \angle PCO$.

260. ANOTHER TRIANGLE FALLACY

To prove that the sum of two sides of a triangle is equal to the third side.



Let ABC be a triangle.

Complete the parallelogram and divide the diagonal AC into n equal parts. Through the points of division draw $n-1$ lines parallel to AB. Similarly, draw $n-1$ lines parallel to BC. AB will be divided into n equal parts. Also BC will be divided into n equal parts. The parallelogram is now divided into n^2 equal and similar parallelograms.

NOTE. — The diagram is drawn for $n = 3$.

Taking the small parallelograms of which the segments of AC are diagonals, we have

$$AB + BC = AM + EF + GH + ME + FG + HC.$$

A similar relation is true, however large n may be. Now let n increase indefinitely. Then the lines AM, ME, EF, etc., will get smaller and smaller. Finally the points MFH will approach indefinitely near the line AC, and ultimately will lie on it. When this is the case the sum of AM and ME will be equal to AE, and similarly for the other similar pairs of lines.

$$\text{Hence, } AM + ME + EF + FG + GH + HC = AE + EG + GC,$$
$$\text{or } AB + BC = AC.$$

THE FOURTH DIMENSION

Geometry as studied in the schools is divided into two parts, Plane Geometry, or Geometry of Two Dimensions, and Solid Geometry, or Geometry of Three Dimensions. These divisions naturally suggest an infinite number of divisions. Considering space as an aggregate of points, the line is a one-dimensional space, a plane is a two-dimensional space, and a solid is a three-dimensional space. To fix exactly the position of a point on a line, it is only necessary to have one number giving its distance from some fixed point. To fix exactly the position of a point in a plane, it is necessary to start from a known point and measure in two given perpendicular directions. To fix exactly the position of a point in a solid, it is necessary to start from a known point and measure in three perpendicular directions.

Thus to locate a man traveling north from a given place it is necessary to know only the distance traveled. To locate a man traveling on the sea it is necessary to have two measurements given — his latitude and longitude. To locate a man traveling in the air it is necessary to have three measurements given — his latitude, longitude, and his distance above or below the sea level.

The question now arises: Why may there not be a space of four dimensions and thus a geometry of four dimensions in which the exact position of a point may be determined by measuring in four perpendicular directions? This question is one which we cannot escape. Paul may have had the fourth dimension in mind, when, speaking of spiritual life, he said, "That Christ may dwell in your hearts by faith, that ye being rooted and grounded in love, may be able to comprehend with all saints what is the breadth, and length, and depth, and height" (Eph. 3:17, 18); or when he wrote, "I knew a man whether in the body, or out of the body, I cannot tell, how that he was caught up into paradise and heard unspeakable words"

(2 Cor. 12 : 2, 3). What did John mean when he "was in the spirit viewing the Heavenly Jerusalem" and said, "The city lieth foursquare" (Rev. 21: 16)? Was Christ's transfigured body a four-dimensional body? Was his resurrected body which appeared in the midst of a closed room a four-dimensional body? Was the ascension a like disappearance?

Although these questions cannot be answered by man, we are certain that the term fourth-dimensional came to us from a firm believer in spiritual life. We can neither prove nor deny its existence. If a physical fourth dimension exists, a three-dimensional body would never know it, nor would we have any way of finding out.

If we connect all points of our space, a three-dimensional space, with an assumed point outside of it, then the aggregate of all the points of the connecting lines constitutes a four-dimensional space, or hyperspace. As a moving point generates a line, as a line moving outside itself generates a surface, as a surface moving outside itself generates a solid, just so a solid moving outside of our space would generate a hypersolid, or portion of hyperspace. Hyperspace itself may be conceived as generated by our entire space moving in a direction not contained in itself, just as our space may be generated by the similar motion of an unlimited plane.

Has hyperspace a real, physical existence? If so, our universe must have a small thickness in the fourth dimension; otherwise, as the geometrical plane is assumed to be without thickness, our world, too, would be a mere abstraction (as, indeed, some idealistic philosophers have maintained), that is, nothing but a shadow cast by a more real fourth-dimensional world.

Of what use is the conception of hyperspace? It is of importance to the mathematician. The notion of such a geometry as a logical system of theorems involved in a set of axioms is important to the student. It gives a deeper insight into geometry. The conception of space to which these geometries apply is of great assistance in the application of geometry to the other mathematics. Especially is it of importance because of the parallelism between algebra and geometry. It has very appropriately been called the playground of mathematics. It is not only of importance to the mathematician, but is also of much importance to the philosopher, psychologist, and scientist in general. It is a question of interest to every person.

The geometry of two dimensions is more extensive than the geometry of one dimension. Also the geometry of three dimensions is more extensive than the geometry of two dimensions, yet nearly everything in the solid is more or less analogous to something in the plane. Just so geometry of four dimensions would be still more extensive than geometry of three dimensions, yet very closely related to it. For example, the circle studied in a geometry of one dimension has very few properties, while studied in a geometry of two dimensions has a center, radii, chords, tangents, etc., and studied in a geometry of three dimensions has further numerous geometrical relations with the sphere, cone, cylinder, etc.

Let us conceive of a space of but one dimension. A being in such a space would be limited to a straight line, which he would conceive as extending infinitely in both directions. If you were a point and lived on a straight line you would be a one-dimensional man. You could not move in two-dimensional space, but could think about it. If you were in two-dimensional space you would never know it. You could move backward and forward only. You could not look up or down, nor from side to side. You could see only the back of the man's head in front of you. You could never turn around and talk to a man behind you. If you encountered another being, neither could pass the other.

Conceive of a world of but two dimensions inhabited by two-dimensional beings. Such a world would lie in a single plane, having length and breadth, but no thickness. The inhabitants of this region might be thought of as the shadows of three-dimensional beings. By a miracle one of these beings becomes endowed with a knowledge of three dimensions. He could then do marvelous things in the eyes of his neighbors. He could disappear and reappear at will. The strongest prison could not hold him. By moving out of the plane in which he lives he could look down into the dwellings and even into the insides of his neighbors. He would then be a god in the presence of the inhabitants of flatland, or shadowland.

If you lived on a surface, you would be a two-dimensional man. You would have no thickness. You could slide around like quicksilver. You would be a flat man and could not understand how a third dimension could possibly exist. You could pass your neighbors. You would be living in a three-dimensional world and never know it. You could pass through a three-dimensional being and never know it. You could pass through a brick wall and never see it. You could not move in three-dimensional space, but could think of it. Only a square or circle would be necessary to imprison you. You could see all around you but could not look down or up. If imprisoned, a being in our space by lifting could liberate you and, to your friends, you would have made a miraculous escape. If you should attempt to imprison a three-dimensional criminal in your two-dimensional jail, he would escape by stepping over the walls of your prison and you would never realize how he eluded you.

Now, if there be a four-dimensional world, our three-dimensional space must lie in its midst. All people would then be three-dimensional shadows of four-dimensional beings. We could only become endowed with four-dimensional knowledge or become four-dimensional beings by supernatural means. We could move in a four-dimensional being, and not understand how such a thing is possible. If there be such a thing as a four-dimensional being, it would perhaps assist us in understanding the following scripture, "That they should seek the

Lord, if haply they might feel after him, and find him, though he be not far from every one of us: for in him we live, and move, and have our being" (Acts 17: 27, 28).

If you were a four-dimensional creature, no three-dimensional prison would hold you, and we should never know how you made your escape. You could take money from a locked safe without opening the door. You could place a plum within a potato without breaking the peeling. You could fill a completely inclosed vessel. You could turn a hollow rubber ball inside out. You could remove the contents of an egg without puncturing the shell, or drink the wine from a bottle without drawing the cork.

EXAMINATION QUESTIONS

ARITHMETIC

TEACHERS' EXAMINATION QUESTIONS. — TEXAS

1. Write the analysis of each of the following:

(*a*) A boy has 75 cents, with which he can buy 5 melons. Find the average price of a melon.

(*b*) A boy has 75 cents, with which he buys melons at the average price of 5 cents each. How many melons does he buy?

2. A trader bought a plantation at $14 per acre, and sold it for $15,824, gaining $2 per acre. Find the cost.

3. Find the product of the smallest prime number greater than 153, and the greatest composite odd number less than 230.

4. From the sum of $29\frac{3}{7}$ and $42\frac{5}{8}$, take the difference of $20\frac{1}{6}$ and $10\frac{3}{16}$.

5. The product of two factors is $\frac{3}{16}$; one of the factors is $\frac{2}{3}$. Find the other.

6. What per cent is gained by buying wheat at $62\frac{1}{2}$ cents per bushel and selling at $67\frac{1}{2}$ cents?

7. In a proportion the inverse ratio of the first term to the second term is $3\frac{1}{5}$; the fourth term is 160. Find the third term.

8. Give solution and analysis: Find the present worth and true discount of a note for $135.75, due 1 year 8 months 15 days hence, money being worth 8 %.

9. What may X offer for a house which pays $895 rent per year that he may receive 8% interest on the investment?

10. Reduce to lowest terms: $.66\frac{2}{3}$; .125; $.37\frac{1}{2}$.

TEACHERS' EXAMINATION QUESTIONS. — OHIO

1. Define aliquot part, mean proportional, maker of a note, denominate number.

2. (*a*) Give the table of liquid measure; of dry measure.

(*b*) How many cubic inches in a dry quart? in a liquid quart?

3. A man bought a lot 8 rods square at the rate of $1000 an acre. He fenced it in at an average cost of 35 cents a yard. He then sold the lot through an agent for $750, paying $2\frac{1}{2}$% commission. Find the man's profit.

4. (*a*) What is meant by "paying by check"?

(*b*) Suppose that you sell to Charles Ray a horse for $250 and agree to give him 5% off for cash. You receive in payment his check for the amount on some bank of which you know. Write the check, supplying the necessary details, but using a fictitious name.

(*c*) How could this check be transferred to another person so that the money could be drawn only on his order?

5. (*a*) A man wishes to build a house 28 feet by 32 feet. He needs four sills, each 6 inches by 8 inches, to put under the walls. How much will they cost at $18 per M?

(*b*) How many feet of siding are necessary for this house, supposing it to be 18 feet high, the siding being 5 inches wide and laid 4 inches to the weather, no allowance being made for gables, doors, or windows?

6. (*a*) A certain district contains taxable property valued at $150,000. The board of education has built a schoolhouse

costing $1800. What will the schoolhouse cost a taxpayer whose property is valued at $4800?

(*b*) Express a tax rate of one mill as a rate per cent.

7. Write a rule for finding (*a*) the area of a circle when the radius is given; (*b*) the surface of a sphere when the radius is given; (*c*) the volume of a pyramid.

8. A father gave his son his promissory note for $225, due when the son became 21 years old. The rate of interest was 5%, and when the note became due, the principal and interest together amounted to $303.75. How old was the son when the note was given?

STATE CERTIFICATE. — KENTUCKY

1. Given the dividend, quotient, and remainder, how may the divisor be found? If 10 apples be divided equally among five boys, which of the terms in the division are concrete and which abstract?

2. What term is the base (*a*) in commission? (*b*) in insurance? (*c*) in profit and loss? (*d*) in interest? (*e*) in discount?

3. At 6 o'clock A.M. the thermometer indicated 20° above zero; at 12 o'clock M., 5° above zero; at 6 o'clock P.M., 7° below zero. Find the average temperature from the three observations. Explain the process.

4. The sum of two numbers is $147\frac{1}{6}$, and their difference $83\frac{1}{4}$. What are the numbers?

5. If equal sums be put at interest for 1 year 12 days, at $5\frac{1}{2}$% and 7% per annum, the difference in interest received on the two principals will be $7.65. Find the sum invested in each case.

6. Wheat is worth 90 cents per bushel, and a field yields 21 bushels per acre, at a cost of $16.75 per acre for cultivation.

If the cost of cultivation be increased 20 %, and the yield be thereby increased 30 %, what is the net gain per acre?

7. The longitude of Pensacola, Fla., is 87° 15′ West. Find the difference between standard time and local (Meridian) time in that city.

8. The proceeds of a 3 months' note discounted at bank at 6 % per annum, the day it was made, were $ 400. Find the face of the note.

9. A contractor in building two residences finds that the number of mechanics employed on the first is to the number employed on the second as 7 : 4, the weekly wages paid individuals on the first to those on the second as 8 : 7, and the time each mechanic was employed on the first to that on the second as 5 : 12. Find the relative cost of labor on the two buildings.

10. How many trees planted 33 feet apart will be required to cover 10 acres in the shape of a rectangle 20 rods wide, if no allowance is made for space beyond the outside rows?

STATE EXAMINATION. — MICHIGAN

1. (*a*) 9 is a factor of a number if it is a factor of the sum of its digits, and not otherwise. Prove.

(*b*) At what time between 2 and 3 o'clock are the minute and hour hands at right angles to each other?

2. In a circle 1 mile in diameter three circles are inscribed, tangent to one another and touching the larger circumference. What is the area of the space inclosed by the three circles?

3. Which would be the better investment and how much better for a capital of $5000: Baltimore & Ohio Railroad stock quoted at $127\frac{7}{8}$, brokerage $\frac{1}{8}$ %, paying semiannual dividends of $3\frac{1}{2}$ % and the balance in a savings bank paying 3 %, or the whole in a 6 % mortgage?

4. Write a concrete problem involving cube root and solve in full as you would require your pupils to solve.

5. Discuss briefly as to the advisability of teaching in the grades: metric system, compound proportion, equations, cube root, geometrical constructions, partnership, longitude and time.

6. A train weighing 126 tons rests on an incline and is kept from moving down by a force 1500 pounds. What is the grade?

7. Change 4321 from scale of 10 to scale of 8 and explain.

8. Find the ratio of the side of a cube to the radius of a sphere if the volume of the cube is twice that of the sphere.

9. Discuss and illustrate graphic arithmetic.

10. The marbles in a box can be divided exactly into groups of 17, but when divided into groups of 16, 18, or 24, 9 remain in each case. How many marbles are there?

County Examination. — Texas

1. Two thirds of A's money equal $\frac{3}{4}$ of B's. If they put their money together, what part of the whole will A own?

2. $ 600.00 Dallas, Texas, Jan. 15, 1904.

For value received I promise to pay David Dooley, or order, on demand, six hundred dollars, with interest at 8 % per annum.

What amount will pay the above note Aug. 20, 1904, at *exact* interest?

3. If you double the rate and time, what must be done to the principal, that the interest be unchanged? How many terms are involved in interest? At what rate must any principal be placed to make 5 times itself in 3 years?

4. *A* is in 40° W. longitude. When it is 3 A.M. at *A*, where must *B* be in order that it may be 10 P.M.?

5. If 16 men hoe 200 acres of cotton in 15 days of 8 hours each, how many boys can hoe 150 acres in 12 days of 6 hours each; provided, that while working a boy can do only $\frac{4}{5}$ as much as a man, and that the boys are idle $\frac{1}{6}$ of the time?

6. A miller charges $\frac{1}{9}$ toll for grinding corn. How many bushels, pecks, and quarts must a man take to mill in order that he may obtain 13 bushels of meal?

7. The solid contents of a cube and of a sphere are each 3,048,625 cubic inches. Which has the greater surface, and how much greater?

8. The ice on a circular lake is $1\frac{1}{2}$ feet thick. If the lake is 1000 yards in circumference, how many cubic feet of ice on the lake?

9. I bought two houses for $ 1800, paying 25 % more for one than for the other. I sold the cheaper house at a profit of 20 %, and the higher priced house at a loss of $16\frac{2}{3}$ %. How many dollars did I gain or lose? What was my gain or loss per cent?

10. A bookseller buys a book whose catalogue price is $ 4 at a discount of 25 %, 20 %, and $8\frac{1}{3}$ %, and sells it at 10 % above the catalogue price. What per cent profit does he make?

COMMERCIAL ARITHMETIC. — INDIANA

1. Illustrate *checking results* by 9's and 11's.

2. A farmer wishes to construct a square granary 18 feet on each side that will hold 800 stricken bushels. Find the depth of the bin by the approximate rule.

3. Illustrate a calculation table.

4. A man had 6 acres of land; to one party he sold a piece 25 rods by 20 rods, and to another party 140 square rods. What per cent of the field remained unsold?

5. Define the following: *Discount series, gross price, net price.*

6. Make a copy of a bill of goods showing the purchase of four articles, one article at a discount of 5 %; the second article, 10 %; the third article, 15 %; the fourth article, 20 %.

7. Illustrate a cost key and also a selling key.

8. A note for $ 1500, dated Jan. 1, 1906, bearing interest at 6 %, had payments indorsed upon it as follows: March 1, 1906, $ 250; July 1, 1906, $ 25; Sept. 1, 1906, $ 515; Nov. 1, 1906, $ 175. How much was due upon the note at final settlement, April 1, 1907?

STATE CERTIFICATE. — OHIO

1. The sum of two numbers is 546, their G. C. D. is 21, and the difference of the other two factors is 8. Find the numbers.

2. At what two times between 4 and 5 o'clock are the minute and hour hands of a clock equally distant from 4?

3. Certain employees, having a 9-hour day, strike because of a proposed reduction of 10 % in wages. They resume work at the same wages, but have a longer day. If the increase in time is (to the firm) equivalent to the proposed cut of 10 %, by what per cent are the hours increased?

4. A dealer sells an article at a gain of 10 %; had he paid for it $16\frac{2}{3}$ % less, and sold it for 7 cents less, he would have gained 25 %. Fnd the cost.

5. A man agrees to pay $ 6000 for a lot in three equal payments, including 6 % interest on unpaid money. What is the yearly payment?

6. A lady buys 20 yards of cloth for $ 20; for some she pays $\frac{1}{4}$ of a dollar a yard, for some $\frac{1}{2}$ of a dollar a yard, and for the remainder $ 4 a yard. How many yards of each kind did she buy, provided she bought a whole number of yards of each?

7. A board is 6 inches wide at one end and 18 inches wide at the other end. If it is 16 feet long, how far from the shorter end must it be cut, parallel to the ends, to divide it into two equal parts?

8. A man has a square tract of land which contains as many acres as it requires rails to build a fence around it. If the fence is four rails high, and the rails are 12 feet long, how many acres are in the field?

9. Pure ground mustard contains 35% of oil. A sample of mustard is adulterated with wheat flour. The per cent of oil found in a sample is 15. Find the per cent of wheat flour in the mixture, allowing 2% of oil to exist naturally in wheat flour.

10. The true discount of a certain sum for one year is $\frac{15}{16}$ of the interest. Find the rate.

TEACHERS' EXAMINATION. — MISSOURI

1. A dealer bought two horses at the same price. He sold one, at a profit of 20%, for $102. The other he sold at a loss of 10%. How much did he receive for the latter?

2. A rectangular aquarium is 32 inches long, 24 inches wide, and 16 inches deep. How many goldfish may be kept in it, allowing 1 gallon of water per fish?

3. A man left St. Louis and traveled until his watch was 1 hour and 3 minutes slow. How many degrees had he traveled and in what direction?

4. The base of a triangular field is 360 yards, and the altitude is 615 feet. How many acres does it contain?

5. Two metal spheres of the same material weigh 1000 pounds and 64 pounds respectively. The radius of the second is 1 foot. Find the radius of the first.

6. A dealer sold an automobile for $ 1000, receiving $ 400 in cash and a note for the rest, due in 3 years, interest 6 %, payable semiannually. How much interest was paid on the note?

7. Which is the better investment and how much, 5 % bonds at 110 or 6 % bonds at 118?

8. Name some subjects given in arithmetic that you think might be properly omitted. Give reasons for your answer.

9. What must be invested in railroad $4\frac{1}{2}$ % bonds at $91\frac{5}{8}$ % to yield an annual income of $ 1350, brokerage at $\frac{1}{8}$ %?

10. Analyze: $\frac{5}{6}$ of the price paid for a cow was $\frac{2}{9}$ of the cost of a horse. The horse cost $ 99 more than the cow. Find the cost of each.

STATE EXAMINATION. — NEW YORK

1. What rate per cent of profit will a man make by paying $ 17.10 for an article, with discounts of 20 %, 10 %, and 5 % from the list price, if he sells it at the list price?

2. Find (*a*) the ratio of the areas of two similar rectangles, the length of one being 36 rods and that of the other 90 rods; (*b*) the ratio of the volumes of two similar spheres, the diameter of one being 6 feet and that of the other 8 feet. State the principle applied in each case.

3. A tank to hold 100 barrels can be only 5 feet wide and $4\frac{1}{2}$ feet deep. What is the required length?

4. If to alcohol which cost $ 1.25 a quart 20 % of its volume of water is added, what will be the rate per cent of profit if the mixture is sold at $ 1.40 a quart?

5. If a certain fraction is increased by $\frac{1}{4}$ of itself, the result multiplied by $\frac{2}{5}$ and the product divided by $\frac{5}{6}$, the reciprocal of the result will be $4\frac{7}{17}$. Find the fraction.

6. Using the mercantile rule, find the amount due May 18, 1907, on a note for $ 650, given Nov. 30, 1903, on which the following payments have been indorsed: Jan. 12, 1905, $ 225; April 23, 1906, $ 250. (Use legal rate of interest.)

7. Determine the number of rods around a square field, the diagonal of which is 340 rods.

8. A man has an income of $ 1925 for an investment in United States Steel stock paying 7 %, purchased at 107, brokerage $\frac{1}{8}$. How does this income compare with that of the same sum invested in a real estate mortgage paying 5 %?

9. If $ 260 placed at interest for 1 year 6 months and 20 days at 6 % produces $ 24.27 interest, what sum placed at interest for 11 months and 24 days at 7 % will produce $ 20 interest? (Solve by proportion.)

10. With no allowance for waste, how many feet of lumber, board measure, will it take to make a watering trough 18 feet long, $2\frac{1}{2}$ feet wide, and 20 inches deep, outside measurements, with lumber $1\frac{1}{2}$ inches thick?

County Examination. — Ohio

1. Explain the meaning of the following: *notation, composite number, insurance premium, commission merchant, trade discount.*

2. If A cuts $2\frac{1}{2}$ cords of wood in $7\frac{1}{2}$ hours, and B $3\frac{1}{4}$ cords in $8\frac{2}{3}$ hours, how long will it take the two together to cut enough wood to make a pile 170 feet long, 4 feet wide, and 6 feet high?

3. (*a*) In the expression "3 % stock at 75," explain fully what is meant. (*b*) Make and solve a problem to show clearly the difference between true discount and bank discount.

4. A person owns $ 15,000 bank stock paying 5 %, which he sells. He invests the proceeds in 6 % stock at 120, his

income being increased $ 60. Find the price at which he sold the first stock.

5. The side of a square inscribed in a circle is 10 feet. Find both the diameter and area of the circle.

6. A miner sold 2 pounds of gold dust at $ 220 a pound avoirdupois, and the broker sold it at $ 16 per ounce Troy. Did he gain or lose, and how much?

7. Write a rule for finding the area of a rectangle, and illustrate by a diagram that children can understand.

8. A man owns a house valued at $ 1500, land valued at $ 2100, and has $ 1500 in a savings bank. If he owes $ 900 and the tax rate is 18 mills, what is the amount of his tax?

COUNTY EXAMINATION. — TEXAS

1. There are two general methods of performing subtraction. Explain the method you use and justify its use.

2. Explain as you would to a class that a fraction may be considered a problem in division.

3. How was the length of the meter determined? The weight of the gram? The capacity of the liter?

4. Nine men can do a work in 8⅓ days. How many days may 3 men remain away and yet finish the work in the same time by bringing 5 more with them?

5. How many square inches in one face of a cube which contains 2,571,353 cubic inches?

6. Find the sum whose true discount by simple interest for 4 years is $ 25 more at 6 % than at 4 % per annum.

7. Find the length of a minute-hand whose extreme point moves 4 inches in 3 minutes 28 seconds.

8. A, B, and C dine on 8 loaves of bread; A furnishes 5 loaves; B, 3 loaves; and C pays 8 cents for his share. How must A and B divide the money?

9. Bought bonds at 12% premium and sold them at a loss of $12\frac{1}{2}$%. At what discount were they sold?

10. (*a*) At what discount should 7% bonds be bought to make 8% on the investment?

(*b*) At what premium should 8% bonds be bought to realize $6\frac{2}{3}$% on the investment?

TRAINING CLASS CERTIFICATE. — NEW YORK

1. Distinguish between the simple and the local value of a figure. How much greater is the local value of 8 in the fourth order of units than in the second decimal place?

2. A student paid $\frac{1}{5}$ of his yearly allowance for books and $\frac{3}{10}$ of the remainder for clothes; he paid $20 more for clothes than for books. What was his yearly allowance?

3. The earth removed in excavating a cellar 33 feet wide and 55 feet long, to a depth of 6 feet, is used to raise the surface of a lot containing $\frac{1}{8}$ of an acre. How much is the surface of the lot raised?

4. It is 9 A.M. at a place 18° 30′ east of New York. What is the time at a place 46° 15′ west of New York? Give a model explanation.

5. The net proceeds of a shipment of 500 tons of hay was $6790 after a commission of 3% had been deducted. What was the selling price per ton?

6. If 46% of the enrollment of a school is boys and there are 162 girls, how many boys are enrolled? Analyze.

7. Give a clear explanation of the process of finding, by factoring, the lowest common multiple of 78, 195, 117.

8. Describe a lesson to develop the table of square measure.

FOR SECOND GRADE CERTIFICATE. — MICHIGAN

1. (*a*) What is the least number by which $\frac{8}{9}$, $\frac{9}{10}$, and $\frac{5}{6}$ can be multiplied to give, in each case, an integer for a product?

(*b*) Divide some number selected by yourself into integral parts having the ratios of $\frac{1}{2}$, $\frac{3}{4}$, and 3, respectively.

2. (*a*) What is the volume in cubic inches of a body that weighs 10 pounds in air and 8 pounds in water?

(*b*) The specific gravity of cork is .24, of gold is 19.36. How much gold can be kept from sinking by a cubic foot of cork?

3. A can do as much work in a day as B in $1\frac{1}{2}$ days. If A can do a piece of work in 12 days, how long for them to do the work together?

4. Sold two horses at $ 120 each. On one I lost 25 %, on the other I gained enough to retrieve this loss. What per cent did I gain?

5. When a certain number is divided by 45 there is a remainder of 30. What would be the remainder if the number were divided by 9?

6. Give the following tables, using proper abbreviations: linear measure, square measure, liquid measure, and avoirdupois weight.

7. Mr. Charles Brown has a note for $ 250 at 6 % interest per annum, running two years, which was given in Detroit $15\frac{1}{2}$ months ago to John R. Clark and by Clark properly indorsed to Brown. Draw the note, making proper indorsement and find the interest due to-day.

8. Analyze: Ten per cent of a consignment of eggs were broken. At what per cent advance must the remainder be sold to realize a gain of 25 %?

9. Formulate and solve an example in both simple and compound proportion.

10. Illustrate in a township the following described parcel of land and find its value at $12.50 per acre: N. $\frac{1}{2}$ of N. E. $\frac{1}{4}$ of S. E. $\frac{1}{4}$, sec. 16.

11. Define (*a*) multiple, (*b*) factor, (*c*) cancellation, (*d*) decimal fraction, (*e*) abstract number, (*f*) ratio, (*g*) percentage, (*h*) per cent.

12. Give principles upon which the following operations are based: (*a*) reducing fractions to lower terms, (*b*) reducing fractions to a common denominator, (*c*) pointing off in multiplication of decimals, (*d*) dividing percentage by rate to find the base.

13. At $2.50 per rod what will it cost to fence a square field containing 10 acres?

14. A jobber retails at a gain of 25% and discounts this price at 20% and 10% for cash. What per cent are his profits on cash sales?

ADVANCED ARITHMETIC. — NEW YORK

1. State *three* principles of the Roman notation and illustrate each. Mention *two* common uses of this system and *two* advantages that the Arabic system has over the Roman.

2. Subtract 6589 from 14,523 and prove the correctness of your result by the method of (*a*) casting out 9's, (*b*) summing up the digits (unitate method).

3. Using the contracted method, find the product of .134567 and 8.4032 correct to *four* places of decimals.

4. If 18 men can do a piece of work in 24 days, in how many days can 27 men do the work? Solve by (*a*) analysis, (*b*) proportion.

5. If the price of milk rises from 6 cents to 9 cents a quart, what per cent is the advance? If the price falls from 9 cents to 6 cents, what per cent is the fall? Explain in full.

6. A boat travels 15 miles downstream in $2\frac{1}{2}$ hours; the boat's rate of travel in still water is $4\frac{1}{2}$ miles an hour. In what time can the boat return? Write analysis in full.

7. A grocer has defective scales which indicate $\frac{1}{2}$ ounce less to the pound than the true weight. What is the value of the tea that he sells for $16.64? Write analysis in full.

8. The exact interest on a debt for a given number of days and at a given rate is $9.25. What would be the interest on the same debt for the same time and at the same rate if computed by the 6 % method? Explain.

TEACHERS' EXAMINATION. — INDIANA

1. Bought 240 barrels of apples at $1.75 a barrel; lost 40 barrels through frost. At what price a barrel must I sell the remainder to gain 25 % on the money invested?

2. Find cost of stone wall 4 rods long, 5 feet high, and 2 feet thick, at 60¢ a square foot.

3. Simplify the following:

$$\frac{3\frac{1}{2} + 2\frac{1}{3} - 1\frac{1}{4}}{\frac{3}{2} \times \frac{10}{3}} \div 1.375.$$

4. A resident of the city, giving up his lease on a house at $30 per month, bought a lot at $1200 and built a house costing $2400. Taxes per year are $56.70; cost of insurance $10, and cost of repairs $25. Allowing interest at 6 % on the amount in the property, how much does he save annually by owning his own property?

5. After wheeling $12\frac{1}{2}$ miles, a boy found he had traveled $83\frac{1}{3}$ % of the distance he had intended to go. How long a ride did he expect to take?

6. The wheels of a locomotive are 15 feet 5 inches in circumference and make 8 revolutions a second. How long does it take it to run 100 miles?

7. Central Park, New York, contains 879 acres, and the new reservoir in the Park contains 107 acres. What per cent of the park does the reservoir cover?

8. Find the interest on $1150 for 1 year 3 months and 17 days at 6 %.

COUNTY EXAMINATION. — TEXAS

1. Three boys had 169 apples which they shared in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. How many did each receive?

2. What is the difference in area between a half of a foot square and half of a square foot?

3. A man living in Galveston observed that his clock, correct by sun time, was 19 minutes slower than the depot clock, correct by standard time, 90th meridian. Find longitude of Galveston.

4. A merchant bought cloth at $1.15 per meter and sold it by the yard at a profit of 20 %. How much did he get per yard?

5. The distance from Austin to San Antonio is 152,064 varas. Find the distance in miles.

6. A merchant paid $1323 for goods, and the discounts were 25 %, 12$\frac{1}{2}$ %, and 10 %. Find the list price.

7. An agent sells 1200 barrels of apples at $4.50 a barrel and charges 2$\frac{1}{2}$ % commission. After deducting his commission of 8 % for buying, he invests the net proceeds in cotton. What is his entire commission?

8. How much must be invested, if stock 20 % below par yield a 6 % income of $390?

9. How large a draft, payable in 30 days after sight, can be bought for $352.62, exchange 1$\frac{1}{2}$ % discount, and interest at 6 %?

10. A grocer has a false balance which gives $14\frac{1}{2}$ ounces to the pound. What does he gain by the cheat in selling sugar for $ 258.56?

11. What would be the cost of 10 planks each 18 feet long, 15 inches wide, 2 inches thick, at $ 40 per thousand board feet?

FOR STATE CERTIFICATE. — OHIO

1. A and B run a race, their rates of running being as 17 to 18. A runs $2\frac{1}{3}$ miles in 16 minutes 48 seconds, and B the whole distance in 34 minutes. What is the distance run?

2. The surface of the six equal faces of a cube is 1350 square inches. What is the length of the diagonal of the cube?

3. A man bought 5 % stock at $109\frac{1}{2}$, and $4\frac{1}{2}$ % pike stock at $107\frac{1}{2}$, brokerage in each case $\frac{1}{2}$ %; the former cost him $ 200 less than the latter, but yielded the same income. Find the cost of the pike stock.

4. A, B, and C start together and walk around a circle in the same direction. It takes A $\frac{5}{36}$ hours, B $\frac{2}{9}$ hours, C $\frac{35}{99}$ hours to walk once around the circle. How many times will each go around the circle before they will all be together at the starting point?

5. I hold two notes, each due in two years, the aggregate face value of which is $ 1020. By discounting both at 5 %, one by bank, the other by true discount, the proceeds will be $ 923. Find face of bank note.

6. The hour and the minute hands of a watch are together at 12 o'clock. When are they together again?

7. How many cannon balls 12 inches in diameter can be put into a cubical vessel 4 feet on a side; and how many gallons of wine will it contain after it is filled with the balls, allowing the balls to be hollow, the hollow being 6 inches in diameter, and the opening leading to it containing 1 cubic inch?

8. An agent sold a house at 2 % commission. He invested the proceeds in city lots at 3 % commission. His commissions amounted to $350. For what was the house sold?

For State Certificate. — Tennessee

1. What is the difference between common and decimal fractions?

2. Multiply one tenth by twenty-five ten-thousandths, divide the product by five millionths, and subtract nine tenths from the quotient.

3. When it is 10 o'clock A.M. at Berlin, 13° 23′ 43″ E., what is the time at Boston, 71° 3′ 30″ W.?

4. A, B, and C can together mow a field in 25 days; A can mow it alone in 70 days, and B in 80 days. In what time can C mow it alone?

5. How many gallons of water will a cistern 5 feet in diameter and 10 feet in depth hold?

6. A merchant sold a watch for $40 and lost 20 %. With the $40 he bought another watch, which he sold at a gain of 20 %. What was the merchant's gain or loss by the transactions?

7. Find the annual interest on $560 for 4 years 3 months and 18 days.

8. If 1800 men have provisions to last $4\frac{1}{2}$ months, at the rate of 1 pound 4 ounces a day to each, how long will five times as much last 3500 men, at the rate of 12 ounces a day to each man? (Solve by proportion.)

9. What will it cost, at 90 cents per yard, to carpet a room $19 \times 14\frac{1}{2}$ feet, strips running lengthwise, with carpeting $\frac{3}{4}$ yard wide?

10. How many posts, placing them 8 feet apart, will be required to fence a square field containing 16 acres?

STATE EXAMINATION. — OHIO

1. What fraction is $\frac{4}{9}$ of its reciprocal?

2. The hands of a clock coincide every 66 minutes. How much does the clock gain or lose in one hour?

3. Wishing to know the height of a certain steeple, I measured the shadow of the same on a horizontal plane $27\frac{1}{2}$ feet. I then erected a 10-foot pole on the same plane and it cast a shadow $2\frac{2}{3}$ feet. What was the height of the steeple?

4. A offered me a bill of sugar for $1800 on 6 months' credit, or for the present worth of that sum for cash. I accepted the latter offer and obtained the money at a bank for the same time at 6 %. Did I lose or gain and how much?

5. A stone was thrown into an empty cylindrical vessel, which was then filled with water; when the stone was taken out, the water fell 4.75 inches. What was the volume of the stone, the diameter of the vessel being 9 inches?

6. A passenger train leaves a certain station at 2 o'clock, to go to the end of the road, 120 miles, and travels at the rate of 25 miles an hour. At what time must a freight train which travels at the rate of 15 miles in 50 minutes, have left, so as not to be overtaken by the passenger train?

7. A owns a house which rents for $ 1450, and the tax on which is $2\frac{3}{4}$ % on a valuation of $ 8500. He sells for $ 15,300 and invests in stock at 90 that pays 7 % dividends. Is his yearly income increased or diminished, and how much?

8. The distance between the centers of two wheels is 12 feet. If their radii are 7 feet and 1 foot, find the length of the belting necessary for one to run the other.

FOR STATE CERTIFICATE. — TENNESSEE

1. State the difference between common and decimal fractions.

2. Approximately the longitude of Carthage is 10 degrees 15 minutes and 20 seconds east, while that of Colon is 79 degrees 25 minutes and 30 seconds west. When it is 9 o'clock A.M. at Carthage, what is the hour at Colon?

3. 87 % of 961 is 29 % of what number?

4. Make formulæ for each case of percentage.

5. A boy had two goats which he sold for $6 each. What did they cost him if he gained 20 % on one and lost 20 % on the other?

6. Write a negotiable promissory note; a draft; a check.

7. Find the annual interest on $760 at 5 per cent for 4 years 5 months 18 days.

How long must $84.80 be put on interest at $5\frac{1}{2}$ % to amount to $102.29?

8. Divide 65 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

9. If a mow of hay 32 feet long, 16 feet wide, and 16 feet high lasts 8 horses 20 weeks, how many weeks will a mow of hay 28 feet long, 20 feet wide, and 12 feet high last 5 horses?

10. Two trees, 80 and 120 feet high, respectively, are 30 yards apart. What is the distance between their tops?

TEACHERS' EXAMINATION. — OHIO

1. Find the decimal which when added to the difference of $\frac{9}{200}$ and 0.002775 produces the square of 0.215.

2. A can do a piece of work in 2 hours, B in $2\frac{1}{2}$ hours, and C in $3\frac{1}{2}$ hours. How much of the work can they do in 20 minutes, all working together?

3. Find the principal that will amount to $131.88 in 2 years 11 months 15 days at 6 %.

4. Write an example in trade discount and give solution.

5. Sold an invoice of books at a loss of $16\frac{2}{3}\%$. Had I paid \$400 less, my gain would have been 25%. What was the selling price?

6. A's money added to $\frac{2}{8}$ of B's, which is to A's as 2 is to 3, being put on interest for 6 years at 4% amounts to \$744. How much money has each?

7. I received \$4850 and a consignment of 2000 barrels of flour which I sold at \$7.50 a barrel and invested the net proceeds and cash in cotton. How much did I invest in cotton, my commission being 3% for selling and $1\frac{1}{2}\%$ for buying, and the expenses for storage and freight \$350?

8. What should be paid for a 6% stock that 8% may be realized on the investment?

9. When do the hour and minute hand of a watch coincide between 8 and 9 o'clock?

10. A bushel measure and a peck measure are of the same shape. Find the ratio of their heights.

For County Superintendent. — Tennessee

1. A man has $1\frac{1}{4}$ miles to go; after he has gone

$$\frac{\frac{1}{2} + \frac{2}{5} \times 1\frac{1}{4} - \frac{1}{7}}{\frac{3}{4} \times 1\frac{7}{9} \div \frac{2}{3}}$$

of a mile, how far has he yet to go?

2. Simplify: $(0.08\frac{1}{4} + 1.2\frac{1}{2}) \div (0.006\frac{1}{4} \times 0.016)$.

3. Reduce 2 pecks 3 quarts 1.2 pints to the decimal of a bushel.

4. A man sold two horses for \$200 each. On one he made 50% of the cost, and on the other he lost 50%. Did he make or lose by these sales, and how much?

5. A merchant sends his agent \$10,246.50 with which to buy flour. After deducting his commission of $3\frac{1}{2}\%$, how many barrels of flour at \$5.50 a barrel can be purchased?

6. A note of $850 with interest payable annually at 5% was paid 3 years 3 months 18 days after date, and no interest had previously been paid. What was the amount due?

7. What is the *exact interest* on $600 at 5% for 90 days?

8. If 4 men can dig a ditch 72 rods long, 5 feet wide, and 2 feet deep in 12 days, how many men can dig a ditch 120 rods long 6 feet wide 1 foot 6 inches deep in 9 days?

9. Find the cube root of 28.094464.

10. A man receives $630 as his annual dividend from 7% stock. How many shares of $100 each does he hold.

TEACHERS' EXAMINATION. — GEORGIA

1. What is a Unit? A Number? A pure, or abstract, Number? What is an Integer?

2. At what time should Wentworth's Elementary Arithmetic be taken up? What kind of training ought the child to have had as an introduction to book work?

3. What powers ought to receive special training before book work is begun?

4. Give suggestions of lessons intended to train (*a*) the eye, (*b*) the ear, (*c*) the touch. Would any good purposes be served by having arithmetic lessons relate generally to the community and its life? Why?

5. Change the following numbers in Roman Notation into Arabic Notation:

DXLVI, MCDXCII, CCIV, MDCCCCXI, DCXI.

6. Define the following: a Prime Number; a Composite Number; Factor; Multiple; Least Common Multiple.

7. A farmer who owned $\frac{8}{9}$ of an acre of land sold $\frac{3}{4}$ of his share at the rate of $300 an acre. How much did he get for it?

8. What is Ratio? Proportion? The Washington Monument casts a shadow 223 feet 6½ inches when a post 3 feet high casts a shadow 14.5 inches. What is the height of the monument?

9. A man bought 20 acres of land at $50.25 an acre. He sold ⅓ of an acre to B, 8⅗ acres to C, and the remainder to D. If he received $65 an acre from B and C, and $60 an acre from D, how much did he gain?

10. James McKnight bought from James Laird, Charleston, S.C., as follows:

40 joists 2 × 6, 18 feet long, at $25 per M.
16 beams 6 × 9, 20 feet long, at $30 per M.
72 scantling 2 × 4, 12 feet long, at $24 per M.
240 boards 1 × 10, 12 feet long, at $18 per M.
24 planks 2 × 14, 16 feet long, at $17.50 per M.

Make out complete bill, and find amount due Laird.

For County Certificate. — Louisiana

1. Find the difference between 1⅔ × 2⅝ and 0.019 of 220.

2. Express ratio of 25⅔ yards to 14¾ rods in three different ways: first, as a common fraction in its lowest terms; second, as a decimal fraction; and, third, a rate per cent.

3. On November 21, 1908, Henry Brown loaned to Peter White on his note for 2 years at 8 per cent, $500.

Write the note. Payments on the note were made as follows:

Jan. 1, 1909	$200
Sept. 15, 1909	125

What was due at maturity of note?

4. A real estate dealer asked for a farm 25 per cent more than it cost. He finally took 15 per cent less than the asking price and gained $1000. What was his asking price? (Analyze.)

5. If 4 men dig a trench in 15 days of 10 hours each, in how many days of 8 hours each can 5 men perform the same work? (Analyze.)

6. What will be the cost of a pile of wood 20 feet × 14 feet × 12 feet at $3.50 a cord?

7. A, B, and C enter into partnership. A puts in $500 for 5 months, B puts in $1000 for 8 months, and C $1500 for 2 years. They gain $1200. What is the share of each?

TEACHERS' CERTIFICATE. — FLORIDA

1. A man having 100 fowls sold $\frac{1}{4}$ of them to E and $\frac{2}{3}$ of the remainder to F. What was the value of what remained, if they were worth 26 cents apiece?

2. What is the exact value of $\left(3+2\frac{1}{2}-\frac{3}{4} \text{ of } \frac{6}{2}+\frac{4}{\frac{2}{3}}\right)\div 4\frac{1}{5}$?

3. A man sold 8 bushels 3 pecks 4 quarts of cranberries at $\$3\frac{1}{2}$ a bushel, and took his pay in flour at $3\frac{1}{2}$ cents a pound. How many barrels of flour did he receive?

4. The difference in time between London and New York is 4 hours 55 minutes $37\frac{2}{3}$ seconds. What is their difference in longitude?

5. How much less would it cost to make a brick sidewalk $4\frac{1}{2}$ feet wide and 260 feet long, at $1.08 a square yard, than to lay a stone walk of the same dimensions, at 22 cents a square foot?

6. A merchant marked cloth at 25% advance on the cost. The goods being damaged, he was obliged to take off 20% of the marked price, selling it at $1 per yard. What was the cost?

7. What is the duty on 18 pieces of Brussels carpeting, of 60 yards each, invoiced at 45 cents per yard, the specific duty being 38 cents per yard, and the ad valorem duty 35%?

8. If 9 men can mow 75 acres of grass in 6 days of $8\frac{1}{4}$ hours each, in how many days of 8 hours each can 15 men mow 198 acres?

9. A merchant bought a bill of goods amounting to $3257 on a credit of 3 months, but was offered a discount of $2\frac{1}{2}$ % for cash. How much would he have gained by paying cash, money being worth 7 %?

10. How many cubic feet are there in a spherical body whose diameter is 25 feet?

TEACHERS' EXAMINATION. — CALIFORNIA

Oral Arithmetic

1. I sold a horse for $60 and thereby lost $\frac{1}{4}$ of the cost. What should I have sold it for to gain $\frac{1}{5}$ of the cost?

2. If to a certain number $\frac{1}{2}$ of itself and $\frac{1}{3}$ of itself be added, the sum will be 66. Find the number.

3. A bicyclist rode 27 miles in 2 hours 15 minutes. What was the rate in miles per hour?

4. What is the square of $3\frac{1}{2}$? Answer to be a mixed number.

5. Write equivalent common fractions for the following decimals: $.87\frac{1}{2}$, $.62\frac{1}{2}$, $.06\frac{1}{4}$.

6. A, B, and C enter into partnership. A puts in $400 for 1 year; B $300 for 2 years; C $200 for 4 years; they gain $720. What is the share of each?

7. Sold 24 boxes of apples at $1.50 a box, and bought cloth with the proceeds at $.75 a yard. How many yards did I buy?

8. What per cent of $51\frac{1}{3}$ is $17\frac{1}{9}$?

9. A field containing 3200 square rods is just twice as long as it is wide. What are its dimensions?

10. $3 \div \frac{1}{5} \times \frac{1}{2} + 2\frac{1}{2} - 6$ is $\frac{2}{3}$ of what number?

For State Certificate. — Washington

1. Analyze: A has 20% more money than B, who has 25% more than C. A has $80 more than C. How much has each?

2. Analyze: A can do a piece of work in 13 days, B in 18 days, and C in 20 days. After all have worked 4 days, how long will it take C, working alone, to finish?

3. If the proceeds of a sale of 20 tons of potatoes, allowing 4% commission, was $432, at what price per hundredweight were they sold?

4. Goods marked to be sold at 35% profit, were sold at a discount of 20% from marked price; the gain was $192. What was the marked price?

5. What is the capacity in liters of a tank 4 meters 6 decimeters long, 3 meters 2 decimeters wide, 2 meters 5 decimeters deep? What is the capacity in kiloliters?

6. Principal $675; time 1 year 6 months. Find amount and write the note in full, making it negotiable by indorsement.

7. Find one edge of a cube whose volume is 2515.456 cubic inches.

8. If 24 men in 15 days of 12 hours each dig a trench 300 rods long, 5 yards wide, and 6 feet deep, in how many days of 10 hours each can 45 men dig a trench 125 rods long, 5 yards wide, and 8 feet deep? (Solve by proportion.)

9. (*a*) Find $\frac{3}{4}$ of 3 miles 64 rods 3 yards 2 feet 8 inches.

(*b*) Express .45 mile in integers of lower denominations.

10. Find the number of board feet in four pieces $10'' \times 2' \times 16'$, two pieces $10'' \times 8'' \times 32'$, and one piece $12'' \times 12'' \times 40'$.

11. Find the volume of the largest square prism that can be cut from a cylinder 4 feet in diameter, 12 feet long.

For State Certificate. — Washington

1. Analyze: A horse cost one fourth more than a carriage; the horse was sold for 20 % more than cost, and the carriage for 20 % less than cost. Both together sold for $368. What was the cost of each?

2. Analyze: At what time between 8 and 9 o'clock are the hands of a watch together?

3. When it is 6 P.M. at St. Paul 95° 4′ 55″ west, it is 33 minutes 54 seconds after 1 A.M. next day at Constantinople. What is the longitude of Constantinople?

4. Find the proceeds of note of $825, drawing interest at 7 % per annum, given April 25, 1908, due 6 months after date, discounted July 13 at 8 % per annum.

5. What annual income is derived from $8475 invested in 5½ % bonds bought at 113?

6. (*a*) What number is 40 % more than 850?
   (*b*) 1050 is how many per cent more than 630?
   (*c*) What number is 20 % less than 800?
   (*d*) 600 is 25 % less than what number?
   (*e*) 900 is how many per cent less than 1200?

7. The hypotenuse of a right triangle is 115, its altitude is 92. What is its base? What is its area?

8. The dimensions of a rectangular solid are 24 inches, 20¼ inches, and 12 inches. Find its area and volume. Find the edge of a cube of equal volume.

9. Find the area in hectares of a field 30 dekameters in length, 20 dekameters in width.

10. If the freight on 30 head of cattle, each weighing 1400 pounds, for a distance of 160 miles, is $112, what should be the freight on 36 head, each weighing 1800 pounds, for a distance of 140 miles? (Solve by proportion.)

FOR STATE CERTIFICATE. — OREGON

1. A well at Madison, Wisconsin, furnishes enough water to irrigate 110 acres of land 2 inches deep, every 10 minutes. At this rate how many acres can it cover to the depth of 1 inch every day?

2. A dealer bought two horses at the same price. He sold one at a profit of 20% for $102. The other he sold at a loss of 10%. How much did he receive for the latter?

3. (*a*) Find the interest on $625.20 for 6 months 9 days at 5%. (*b*) Some 4-foot wood is piled 5 feet high. The pile is 2 rods long. How many cords are there?

4. Find the *discount* and *proceeds* of the following note: Face, $175. Time, four months without grace. Rate, 6%.

5. An agent has $590 to invest after deducting his commission of 2% on the money invested. What amount does he invest?

6. The distance around a square farm is 3 miles 240 rods. Find the length of each side; the area in acres.

7. Allowing 231 cubic inches to the gallon, how many gallons in a watering trough that is 6 feet long and 16 inches wide, the ratio of its depth to its width being 3:4?

8. A boy in a grocery store receives $8 a week. He spends 20% of it for board, 20% of the remainder for clothes, and $2 in other ways. If he saves the rest, how much will he save in a year?

9. (By proportion.) When 2 men can mow 16 acres of grass in 10 days, working 8 hours a day, how many men would it take to mow 27 acres in 9 days, working 10 hours a day?

10. How long must a pile of wood be to contain 10 steres, if it is 3.5 meters high and 3.8 meters wide?

11. The diagonal of one face of a cube is $\sqrt{162}$ inches. Find the surface and the volume of the cube.

12. What will it cost to gild a ball 25 inches in diameter at $13.50 a square foot?

EXAMINATION FOR TEACHERS' CERTIFICATE. — PENNSYLVANIA

1. The longitude of Washington, D.C., is 77° 03′ 06″ west. Tokyo is 139° 44′ 30″ east. When it is 6 o'clock P.M. in Washington, Feb. 10, what is the time in Tokyo?

2. How many yards of carpet 27 inches wide are required to cover a floor 20 feet long and 15 feet wide, allowing $5\frac{1}{2}$ yards for matching?

3. On March 9, 1908, John Doe bought a house from Richard Roe for $6000; 20% of the price was paid immediately and a 6-months note bearing 6% interest, given for the remainder. The note was discounted at bank April 9. Write the note and find the discount.

4. Three contractors, A, B, and C, did work for which they received $1500. A furnished 12 men 24 days; B, 20 men 12 days; and C, 18 men 20 days. What is the share of each?

5. How far is it between the tops of two trees which are 80 feet apart, if their heights are 40 feet and 100 feet respectively?

6. The weight of a ball 4 inches in diameter was 8 pounds; $\frac{1}{4}$ of the diameter was turned off. How many cubic inches were turned off, and what was its weight then?

TEACHERS' EXAMINATION. — WASHINGTON

1 A boy, after doing $\frac{3}{8}$ of a piece of work in 30 days, is assisted by his father, with whom he completes the work in 6 days. How long would it have taken each to do the work alone? (Analyze in full.)

2. A fruit dealer bought oranges at the rate of 40 for $1, and sold them at 50 cents per dozen. Find gain per cent. He also bought apples at the rate of 5 for 2 cents and sold them at 8 cents per dozen. How many must he buy and sell in order to gain $2? (Analyze in full.)

3. A farmer finds that a bin 8 feet long, 3 feet 6 inches wide, and 5 feet deep holds about 112 bushels. How many bushels may be contained in a bin 50 % longer, twice as wide, and 50 % as deep?

4. A man who owns a quarter section of coal land claims that he has a bed of coal 6 feet thick covering the entire quarter. If so, how many tons of coal has he, allowing 40 cubic feet to the ton?

5. The steeple of a certain church is a pyramid 28 feet in slant height and stands upon a base 14 feet square. Find cost of painting it at 10 cents per square yard.

6. A certain city bought two horses for the fire department, but finding them unfit for the work, sold them for $300 each, thus gaining 20 % on one, and losing 20 % on the other. Did the city gain or lose, and how much?

7. A rectangular lot contains one acre and has a street frontage of 120 feet. How deep is the lot and how many yards of fence are required to inclose it?

8. The hour hand of a clock is 4 inches long. Over what area does it pass upon the dial during a school day; that is, from 9 A.M. to 4 P.M.?

9. Our most expensive battleship, the *Connecticut*, cost $6,000,000, and the *Louisiana* $97\frac{1}{2}$ % as much. This was 117 % of the cost of the *Vermont*, which cost $\frac{1}{4}$ more than the *Kansas*. Find total cost of this division of our fleet.

10. The cruiser *Olympia* is $21\frac{1}{19}$ % faster than the battleship *Oregon*, which is a 19-knot vessel. If each runs at full speed, how much can the former gain upon the latter in going from Tacoma to Seattle, the distance being 23 knots?

11. A swimming tank is 40 meters long and 15 meters wide. When filled to an average depth of 2 meters, how many liters of water does it contain? Find weight of the water in kilograms.

FOR STATE CERTIFICATE. — COLORADO

1. A man bought a lot for \$1200, and built a house for \$1980. He insured the house for $\frac{2}{3}$ of its value at $\frac{3}{4}$%. The house burned and the lot was sold for \$1328. How much was the gain or loss?

2. At what price must you mark a hat costing \$1.50 so you can discount the price 20% and still make 12%?

3. In a school $\frac{1}{4}$ of the pupils study grammar, $\frac{1}{5}$ arithmetic, $\frac{1}{3}$ geography, and the remainder, which is 39, write. How many pupils in the school?

4. (*a*) How many yards of brussels carpet $\frac{3}{4}$ yard wide will cover a floor 24 feet 9 inches long and $17\frac{1}{2}$ feet wide, if the strips run lengthwise and the matching of the figure requires that 6 inches be turned under? (*b*) What will the carpet cost at \$1.65 per yard?

5. When 5% bonds are quoted at 104, what sum must be invested to yield an annual income of \$800?

6. If 14 persons spend \$1120 in 8 months, at the same rate, what will 9 of the same persons spend in 5 months?

7. \$500.00 Denver, Colo., May 12, 1908.

Ninety days after date, I promise to pay to the order of Charles Taylor, Five Hundred Dollars, at the Central National Bank. Value received.

JOHN J. SMITH.

Discounted May 25, 1908. Find the proceeds. Rate 6%.

8. How many square feet of surface in a stovepipe 16 feet long and 7 inches in diameter?

9. A, B, and C can do a piece of work in 10 days, and B and C can do it in 18 days. In what time can A do it alone?

10. How many board feet in 16 pieces of lumber, each being 14 feet long, 16 inches wide, and $1\frac{1}{2}$ inches thick?

11. The specific gravity of sand is $3\frac{1}{2}$. How much will a cubic yard of sand weigh?

STATE EXAMINATION. — MAINE

1. What are fractions? What names are given to the terms of a fraction? Why are they so named? What is the value of a fraction?

2. Why is it necessary to teach L. C. M. and G. C. D. before teaching fractions? What two things must be taught before teaching L. C. M. and G. C. D.? Find the L. C. M. and G. C. D. of 9, 12, and 54.

3. Add five-sixths, two-fifths, and four-fifteenths. State the four steps taken and give reasons for each.

4. Change 5 shillings and 8 pence to the decimal of a pound. Write the tables of Long and Liquid measures as used to-day. How many cords in a pile of wood 18 feet long, 4 feet wide, and $5\frac{1}{2}$ feet high? Write and solve a problem in Reduction descending.

5. A sold B a farm for $2400, which was 20 % more than it cost him, and took B's note for that amount due in 6 months without interest. If he had that note discounted at a Maine bank, what was his actual gain and what per cent did he gain? Write the note taken. What did A have to do before the bank would discount the note?

FOR STATE CERTIFICATE.—CALIFORNIA

1. I pay $275 for a lot and build on it a house costing $1720, which my agent rents for $25 a month, charging 5 % commission. What per cent do I make on the money invested?

2. A house valued at $1200 had been insured for $\frac{2}{3}$ of its value for 3 years at 1 % per annum. During the third year it was destroyed by fire. What was the actual loss to the owner, no allowance being made for interest?

3. A man purchased goods for $10,500 to be paid in 3 equal installments, without interest; the first in 3 months, the second in 4 months, the third in 8 months. How much cash will pay the debt, money being worth 7 %?

4. The surface of a sphere is the same as that of a cube, the edge of which is 12 inches. Find the volume of each.

5. Subtract $10\frac{9}{24}$ from $15\frac{7}{8}$, divide the remainder by $\frac{1}{4}$, add .625 to the quotient, multiply this sum by $16\frac{2}{3}$, and add $66\frac{24}{36}$ to the product.

6. A square field contains 10 acres. What will it cost to fence it at $1.25 per rod?

7. The longitude of Cincinnati is 84 degrees 26 minutes W., and that of San Francisco 122 degrees 26 minutes 15 seconds W. When it is noon at Cincinnati, what time is it in San Francisco?

8. How many pencils 7 inches long can be made from a block of red cedar 7 inches by $10\frac{1}{2}$ inches by $2\frac{1}{4}$ inches, if the block is sawed into strips $3\frac{1}{2}$ inches wide and $\frac{3}{16}$ inches thick, each strip making the halves of 6 pencils?

9. A man bought a horse for $72, and sold it for 25 % more than cost, and 10 % less than he asked for it. What did he ask for it?

10. A person purchased two lots of land for $200 each, and sold one at 40 % more than cost, and the other at 20 % less than cost, and took a promissory note for the amount of the proceeds of the sale, bearing 8 % interest for 2 years compounded annually. At maturity he collected the note. What per cent of profit was the amount of the note on the original sum invested in the lots.

STATE EXAMINATION. — OKLAHOMA

1. How do you teach the carrying of tens in addition?

2. Illustrate by a drawing of a dial plate that the time past noon plus the time to midnight equals 12 hours.

3. Explain the process of multiplying a fraction by a fraction.

4. Explain the placing of the decimal point in multiplication and division of decimals.

5. Present your method of teaching interest.

6. Factor 1225, 1448, 2356.

7. Illustrate three ways of finding the G. C. D.

8. Find the annual interest on $500 for 5 years 5 months and 5 days at 6 %.

9. The list price of goods is $90. I buy for 20 and 10 off. Find cost to me.

10. The diagonal of a square field is 75 rods. What would be the diagonal of another square field whose area is four times as great? Illustrate.

11. At 66 cents a bushel, what is the value of the wheat which fills a bin 6 feet long and 5 feet square at the ends?

12. The boundaries of a square and circle are each 40 feet. Which has the greater area and how much?

For Third Grade Certificate. — Rhode Island

1. Explain the fact that multiplying the numerator of a fraction or dividing the denominator by a whole number increases the value of the fraction.

2. Simplify the fraction $\left(1\frac{3}{8}+\frac{5}{4}\text{ of }\frac{21}{2\frac{1}{2}}\right)\div 2\frac{77}{114}$.

3. Coffee bought for 20 cents per pound shrinks $8\frac{1}{3}$ %. For how much per pound must I sell it to gain 10 %?

4. Two men are working 8 hours and 10 hours per day at the same daily wages. After working 3 days, each works 1 hour per day more for 3 days. If the amount paid for the whole work is $20.28, what should each receive?

5. Three kinds of tea costing 68 cents, 86 cents and 96 cents a pound are mixed in equal quantities and sold for 90 cents a pound. Find the gain per cent.

6. If a square lot contains 640 acres of land, how many rods of fence will be required to inclose it?

7. The population of a town in 1890 was 12,298, a decrease of $8\frac{1}{3}$ % of the census of 1880; in 1880 there was an increase of $7\frac{1}{2}$ % of the census of 1870. What was the population in 1870?

8. Telegraph poles are usually placed 88 yards apart. Show that if a passenger in a railway train counts the number of poles passed in 3 minutes, this number will express the rate of the train in miles per hour.

9. A gives B a note for $100, payable in 60 days. If B has the note discounted at a bank at 5 % 2 weeks afterward, how much money will he receive?

10. (*a*) If the price of land is $3000 per acre, what would a lot 60 feet by 100 feet cost? (*b*) What would be the cost of a similar lot 50 feet long at double the price?

JUNIOR MATRICULATION. — ONTARIO

1. Express as a decimal:

$$\left(\frac{2.375}{6.\dot{3}} \times \frac{8.\dot{8}}{0.0625}\right) \div \left(\frac{1.7\dot{4}}{12.2\dot{1}} \times 14\tfrac{2}{9}\right) - \frac{5}{6}.$$

2. Use contracted methods to find:

(*a*) $1250\,(1.05)^5$, correct to two decimal places;

(*b*) $1 \div 0.4342945$, correct to four decimal places.

3. How much money deposited in a bank will amount to $1500 in 1 year, the bank paying 3 % per annum, compounded quarterly?

4. A man has a choice of insuring his house for $\frac{3}{4}$ of its value at $1\frac{1}{2}$ %, or for $\frac{7}{8}$ of its value at $1\frac{1}{4}$ %. By what per cent of the value of the house is one premium greater than the other?

5. What is the value of the goods handled in each of the following cases:

(*a*) An agent receives $2450 to invest in goods after retaining his commission of $2\frac{1}{4}$ %?

(*b*) An agent remits to his firm $2450, the proceeds of a sale for which he retains his commission of $2\frac{1}{4}$ %?

6. A man has an annual income of $1785 from an investment in $10\frac{1}{2}$ % stock which is quoted at 137. What would his income be if he had his money out at 7 % interest?

7. What must a Canadian company pay for a draft to cancel a debt of £2430 in London, Eng., exchange being quoted at $8\frac{1}{2}$?

8. The base of a prism of height 125 inches is a parallelogram with a diagonal 104 inches and two sides 45 inches and 85 inches. Find the volume.

9. Find (*a*) the total surface, (*b*) the volume, of a block of wood 18 inches square and 3 inches thick, with a circular hole of 14 inches diameter through its center.

State Examination. — North Dakota

1. Define commission, interest, exchange, annuity.

2. A boy who bought 20 % as many marbles as he had, found that he then had 60. How many had he at first?

3. According to the metric system what is the unit of capacity, of weight, of surface measurements?

4. What must be the length of a plot of ground, if the breadth is 18$\frac{3}{4}$ feet, that its area may contain 56 square yards?

5. What must be the price paid for 5 % stock so that it may yield the same rate of income as 4$\frac{1}{2}$ % stock at 96?

6. A merchant sold a coat for $15.40 and gained 20 %. How much would he have gained if he had sold it for $16.50?

7. What is the depth of a cubical cistern which contains 2744 cubic feet? What will it cost to plaster the sides and bottom at $.35 per square yard?

8. A village must raise $8795 by taxation. The assessed valuation is $989,387, and there are 670 persons subject to a poll tax of $1 each. A's property is assessed at $10,000 and he is a resident of the village. What amount will he pay in taxes?

9. A bridge is 6 rods long and 18 feet wide. What is the cost of flooring this bridge with 3-inch plank at $22.50 per M.?

10. A has $\frac{5}{6}$ more money than B, and together they have $510. How much has each? Give work in full.

For Teachers' Certificate. — Iowa

1. On a map constructed on a scale of $\frac{1}{1600000}$ the distance from Detroit to Chicago is 11.29 inches. How many miles between these cities?

2. What principal will yield $62.50 interest in 1 year 3 months at 4 %?

3. Define: concrete number, interest, gram, date line, cord foot.

4. (*a*) Divide $3\frac{1}{4} - \frac{5}{6} \times \frac{4}{15}$ by $21\frac{1}{5} + \frac{3}{10} + 4\frac{1}{3} \times 5$.

(*b*) What decimal part of a bushel is 2 pecks 4 quarts?

5. What is the area of the circle inscribed in a square whose area is 196 square inches? Of the square inscribed in this circle?

6. A collector has a $500 note placed in his hands with power to compromise; he accepts 75 cents on a dollar and charges 5% of the sum collected, and 25 cents for a draft. What are the net proceeds?

7. What is the difference between local time and standard time at Chicago, the longitude of Chicago being 87 degrees 36 minutes and 42 seconds west?

8. Which is the better discount, 10%, 12%, 5%, or 15%, 6%, 6%? What three equal rates of discount are equivalent to the latter?

9. A cubic foot of water weighs 1000 ounces, and in freezing expands $\frac{1}{40}$ of itself in length, breadth, and thickness. Find the weight of a cubic foot of ice.

10. When a Boston draft for $35,000 can be bought in New Orleans for $34,930, is exchange at a premium, at par, or at a discount? What is the rate?

TEACHERS' STATE EXAMINATION. — IOWA

1. Define: composite number, concrete number, least common multiple of two or more numbers, rectangle, trapezoid, common fraction, decimal fraction.

2. (*a*) Express in Roman notation: 723, 1909, 1776, 2499, 31,749.

(*b*) Express in words: .0276, 100.001, 101, .00047.

3. Reduce 44 rods 5 feet 6 inches to the decimal part of a mile.

4. How many yards of carpet 27 inches wide will be needed to carpet a room 13 feet by 17 feet if the waste in matching is 6 inches on a strip?

5. If goods are bought at 20 and 10 % off and sold at list price, what per cent of profit is made?

6. A note for $ 580 dated March 16, 1909, and due in one year at 6 % interest, was discounted at a bank 3 months later at 8 %. Find the proceeds.

7. A water tank is 16 feet long, 4 feet wide, and $2\frac{1}{2}$ feet high. How many barrels will it hold? How many bushels?

8. Find the number of acres within a circular race track whose circumference is $\frac{5}{8}$ of a mile.

9. A tax of $ 52,000 is to be raised in a city whose assessed property valuation is $ 1,830,000. Find the tax rate. If A's property is assessed at $ 16,000, how much does he pay for his taxes?

10. A factory valued at $ 50,000 was insured for $\frac{3}{4}$ of its value at $\frac{3}{4}$ % premium. Find the annual premium.

11. Find the diagonal of a field that is a half mile long and contains 120 acres. How many rods of fence will be needed to inclose this field?

FOR STATE CERTIFICATE. — SOUTH CAROLINA

1. Divide 7.601825 by 347.512, multiply quotient by .05, to the product add 3.45, and from sum subtract 2.115.

2. Simplify $(3\frac{1}{5} + 4\frac{1}{3} - 5\frac{1}{4} \times \frac{6}{7}) \div (3\frac{1}{2})$.

3. Find the weight in tons of the water in a dock 24 feet deep and covering $\frac{1}{10}$ of an acre, given that a cubic foot of water weighs $62\frac{1}{2}$ pounds.

4. Find the simple interest on $2000 for 2 years 9 months 18 days at 7 %.

5. How many men are required to cultivate a field of $7\frac{7}{8}$ acres in $5\frac{1}{2}$ days of 10 hours each? Given that each man completes 77 square yards in 9 hours.

6. On a map made on a scale of 6 inches to a mile, a rectangular field is represented by a space 1 inch long and $\frac{1}{4}$ inch broad. How many acres are there in the field?

7. At what rate per cent will $2250 amount to $2565 in 4 years at simple interest?

8. If the wholesale dealer makes a profit of 25 % and the retail dealer a profit of 40 %, what is the cost of an article which is sold at retail for $18?

9. What fraction of 39 gallons is 3 bushels and 3 pints? If a gallon contains 231 cubic inches and a bushel contains 2150.4 cubic inches, answer as a common fraction in its lowest terms.

STATE EXAMINATION. — VIRGINIA

1. (*a*) A fruit dealer bought oranges at the rate of 40 for $1, and sold them at 50 cents per dozen. Find gain per cent. (*b*) He also bought apples at the rate of 5 for 2 cents, and sold them at 8 cents per dozen. How many must he buy and sell in order to gain $2? (Analyze in full.)

2. The steeple of a certain church is a pyramid 28 feet in slant height, and stands upon a base 14 feet square. Find cost of painting it at 10 cents per square yard.

3. A certain city bought two horses for the fire department, but finding them unfit for the work, sold them for $300 each: thus gaining 20 per cent on one, and losing 20 per cent on the other. Did the city gain or lose, and how much? (Show work.)

4. A rectangular lot contains one acre and has a street frontage of 120 feet. How deep is the lot and how many yards of fence are required to inclose it?

5. (*a*) What is 16½ % of 900? (*b*) 98 is what per cent of 2450? (*c*) 128 is 32 % of what number? (*d*) 1350 is 25 % more than what number? (*e*) 765 is 10 % less than what number?

6. Find the interest and maturity value of a note of $600 for 3 years 3 months 24 days at 6 %.

7. (*a*) Write a negotiable promissory note, using the above data. (*b*) Make out a bill containing four items of merchandise, and acknowledge payment.

8. A man sold his farm and invested the money at 6 % interest. In one year he spent ½ of his income traveling, ⅛ for a library, and saved $100. Required, selling price of farm. (Analyze in full.)

9. A wagon loaded with hay weighed 43 hundredweight and 68 pounds. The wagon was afterwards found to weigh 9 hundredweight and 98 pounds. Required, value of hay at $10 per ton.

10. What is the net amount of a bill of $800, after allowing successive discounts of 25 %, 10 %, and 5 %?

SECOND CLASS PROFESSIONAL. — ONTARIO

1. Write an article on Arithmetic in Public Schools, under the following headings:

(*a*) Purpose of teaching Arithmetic;
(*b*) Correlation with other subjects;
(*c*) Place and value of Oral Arithmetic.

2. Outline a lesson plan for teaching "8" (Numbers 1–7 are supposed to be known). What facts would you teach before proceeding to "9"?

3. Assuming that your class know how to multiply by a one digit number, show how you would teach the multiplication of 234 by 23.

4. How would you make clear to a class the principles involved in the ordinary method of finding the G. C. M. of such numbers as 2449 and 2573?

5. Mention the topics of all the previous lessons in fractions which you would require to teach as a preparation for a lesson on the multiplication of $\frac{3}{7}$ by $\frac{4}{5}$. Outline your plan for this lesson.

6. Solve, as you would for your pupils, the following:

(*a*) Find the square root of $27\frac{1}{25}$.

(*b*) A man has $6250 6% stock and sells it at 80. With the proceeds he buys a house on which he pays insurance at $\frac{1}{4}$% per annum on $\frac{4}{5}$ of its value, and taxes at 20 mills on the dollar on $4500 assessment, and in addition a water rate of $11 per annum. If he rents the house, what monthly rent should he charge that his annual income may be the same as that derived from the stock?

(*c*) An agent sells 1000 barrels of flour at $5.50 a barrel, and charges $2\frac{1}{2}$% commission; expenses for freight, etc., are $500. With the net proceeds he buys sugar at $6\frac{1}{4}$ cents a pound, charging $2\frac{1}{2}$% commission. How much sugar does he buy?

(*d*) A ditch has to be made 360 feet long, 8 feet wide at the top, and 2 feet wide at the bottom; the angle of the slope at each side being 45°. Find the number of cubic yards to be excavated.

For State Certificate. — North Dakota

1. Define Arithmetic, numeration, compound number, interest, per cent.

2. A farmer sold a horse for $80 and lost 20% of its cost. He then bought a horse for $80 and afterward sold it at a gain of 20%. How much did he gain or lose on the two transactions?

3. Multiply the sum of $\frac{3}{4}$ and $\frac{4}{5}$ by their product and reduce the result to a decimal.

4. Explain the difference between a common and a decimal fraction.

5. The product of three numbers is 420, and two of the numbers are 5 and 7. Find the third number.

6. Find the value of $(\frac{1}{3} + \frac{8}{15}) \times \frac{3}{11} + (\frac{4}{17} + \frac{5}{6} \times 3)$.

7. How many acres in a strip of land 80 rods long and 14 rods wide?

8. C and D together own 921 acres of land, of which C owns 420 acres. C's land equals what fractional part of D's? D's land is what per cent of the whole?

9. What will be the cost of the wood that can be piled in a shed 20 feet long, 10 feet wide, and 8 feet high, at $4.75 per cord?

10. The longitude of Constantinople is 28° 59′ E. When it is noon in Greenwich, what is the time in Constantinople?

TEACHERS' CERTIFICATE. — ARKANSAS

1. A man bought a horse and paid $\frac{1}{3}$ of the price in cash. One year later he paid $\frac{1}{3}$ of what remained, and the two payments amounted to $1530. What was the price of the horse?

2. A having lost 25% of his capital is worth as much as B, who has just gained 15% on his capital; B's capital was originally $5000. What was A's capital?

3. A square field contains 131 acres 65 square rods. What will it cost to fence it at $62\frac{1}{2}$ cents a rod?

4. The width of a river is 100 yards and it averages 5 feet in depth. Find the number of cubic feet of water which flows past a given point in one minute if the average rate of the stream is $2\frac{1}{2}$ miles per hour.

5. A man bought oranges at the rate of 3 for 2 cents, and an equal number at the rate of 4 for 3 cents. He sold them at the rate of 2 for 5 cents and gained $4.30. How many oranges did he buy?

6. A man divided $500 among his three sons, so that the second had $\frac{5}{7}$ as much as the first, and the third $\frac{4}{5}$ as much as the second. How much did each receive?

7. A clock is set at 12 o'clock Monday noon, and on Tuesday morning at 9 o'clock it had lost 3 minutes. What will be the correct time when it strikes 3 o'clock the next Friday afternoon?

8. Find the interest on $9430 for 2 years 5 months 7 days at 5%, using the method which you believe best adapted for class use in teaching interest.

9. The catalogue price of a book is $3. If I buy it at a discount of 40% and sell it at 20% below catalogue price, what is my gain per cent?

10. A and B together have $153; $\frac{3}{4}$ of A's money equals $\frac{2}{3}$ of B's. How much has each? (Write full analysis.)

ONTARIO EXAMINATION QUESTIONS.—UNIVERSITY MATRICULATION

1. From 1870 to 1880, the population of a town increased 30%; from 1880 to 1890 it decreased 30%. The population in 1870 exceeded that in 1890 by 2781. Find the population in 1880.

2. (*a*) A man borrows $12,000 for a year at 8% and loans it at 2% per quarter year, compounding interest at the end of each quarter. How much money will he have made at the end of the year?

(*b*) A borrows from B a sum of money and agrees to pay him by three annual payments of $ 200 each. If money is worth 5 % per annum, compound interest, find the sum borrowed.

3. A commission merchant received 500 barrels of flour, which he sold at $ 5 a barrel, charging 2 % commission; he was instructed to invest the net proceeds, deducting a purchase commission of 2 %, in tea. Find the value of the tea bought, and the total commission.

4. A man holds $ 15,600 stock worth 60; to transfer to 4 % stock at 78 will increase his annual income $ 12; he effects the transfer, but not until each stock has increased 2 in price. Find the increase of his income.

5. A merchant marks his goods at an advance of 25 % on cost. After selling $\frac{1}{3}$ of the goods, he finds that some of the goods in hand are damaged so as to be worthless; he marks the salable goods at an advance of 10 % on the marked price and finds in the end that he has made 20 % on cost. What part of the goods was damaged?

6. A grocer, by selling 12 pounds of sugar for a certain sum, gained 20 %. If sugar advances 10 % in the wholesale market, what per cent will the grocer now gain by selling 10 pounds for the same sum?

7. A note made June 1, at 3 months, was discounted immediately at 8 % per annum, and produced $ 357.40. What was the face of the note?

8. What rate per cent per annum, compounded half-yearly, is equivalent to 6 % per annum, compounded yearly?

9. Two candles are of equal length. The one is consumed uniformly in 4 hours, and the other in 5 hours. If the candles

are lighted at the same time, when will one be three times as long as the other?

10. Calculate the number of acres in the surface of the earth, considering the earth a sphere of 8000 miles diameter.

STATE EXAMINATION.—OHIO

1. I have three pitchers holding respectively $1\frac{1}{2}$, $2\frac{1}{4}$, and $3\frac{1}{2}$ pints. How many times can I fill each from the smallest keg that will hold enough to fill each pitcher an exact number of times?

2. Bought 20 yards cloth, $1\frac{1}{2}$ yards wide, at $ 2 per yard. The cloth shrunk 20 % in length, and 25 % in width. At what price per yard must I now sell the cloth so as to gain 20 %?

3. Bought 6 % railroad stock at $109\frac{1}{2}$, brokerage $\frac{1}{2}$ %. What must the same stock bring 6 years later to pay me 8 % interest?

4. A and B form a partnership. A contributes $ 7000, and is to have $\frac{2}{3}$ of the profits; B contributes $ 3000, and is to have $\frac{1}{3}$ of the profits; each partner is to receive or pay interest at 6 % per annum for any excess or deficit in his share of capital. At the end of the first year the profits are $ 1800. Required worth of each share.

5. How many shares of stock at 40 % must A buy, who has bought 120 shares at 74 %, 150 shares at 68 %, and 130 shares at 54 %, so that he may sell the whole at 60 %, and gain 20 %?

6. A laborer agreed to build a fence on the following conditions: for the first rod he was to have 6 cents, with an increase of 4 cents on each successive rod; the last rod came to 226 cents. How many rods did he build?

7. A wins 9 games of chess of 15 when playing against B, and 16 out of 25 when playing against C. At that rate, how many games out of 118 should C win when playing against B?

8. B agreed to work 40 days at $2 per day, and board; but he agreed to pay $1 a day for board each day that he was idle. How many days was he idle, if he received $44 for his work during the 40 days?

QUARTERLY EXAMINATION. — GUNTER BIBLE COLLEGE

1. Define insurance, arithmetical progression, geometrical progression, and arithmetical complement.

2. What is the distance passed through by a ball before it comes to rest, if it falls from a height of 40 feet and rebounds half the distance at each fall?

3. A merchant adds $33\frac{1}{3}$ % to the cost price of his goods, and gives his customers a discount of 10 %. What profit does he make?

4. What is the difference between the simple and compound interest on $750 for 2 years 7 months, at 5 %?

5. If the duty on linen collars and cuffs is 40 cents per dozen and 20 %, what is the duty on 10 dozen collars at 75 cents a dozen, and 10 pairs of cuffs at 25 cents a pair?

6. The capital stock of a company is $1,000,000, $\frac{1}{4}$ of which is preferred, entitled to a 7 % dividend, and the rest common. If $47,500 is distributed in dividends, what rate of dividend is paid on the common stock?

7. Find the bank discount and proceeds of a 90-day note for $1500 at 6 % interest, dated Aug. 10, and discounted Sept. 1, at 7 %.

8. On Jan. 1, 1908, I borrowed $2000 at 10 % interest, paying $300 every 3 months. I paid the debt in full Jan. 1, 1909. What did I pay by the United States rule?

9. Solve No. 8, by the Merchant's rule. Which method is better for the debtor? Which for the creditor?

10. Given log $2 = 0.3010$, log $3 = 0.4771$, log $5 = 0.6990$.

(*a*) Find log $3^2 \times 5^3$.

(*b*) Find the number of digits in 30^{25}.

EXAMINATION. ARITHMETIC A. — GUNTER BIBLE COLLEGE

1. Define arithmetic, bank discount, specific gravity, involution, commercial discount, and ratio.

2. What is the difference in area between a square whose diagonal is 1 foot and a circle whose diameter is 1 foot?

3. In a lot of eggs 7 of the largest, or 10 of the smallest, weigh a pound. When the largest are worth 15 cents a dozen, what are the smallest worth?

4. My wife's age plus mine equals 76 years, and $\frac{2}{3}$ of her age minus 2 years equals $\frac{1}{2}$ of my age plus 2 years. Find the age of each.

5. The diameter of one cannon ball is $2\frac{1}{2}$ times that of another, which weighs 27 pounds. What is the larger ball worth at 1 cent a pound?

6. Bought apples at $3 a barrel. Half of them rotted. At what price must I sell the remainder in order to gain $33\frac{1}{3}\%$ on the amount bought?

7. Extract the cube root of 926,859,375.

8. A uniform rod 2 feet long weighs 1 pound. What weight must be hung at one end in order that the rod may balance on a point 3 inches from that end?

9. In any year show that the same days of the month in March and November fall on the same day of the week.

10. In a liter jar are placed 1 kilogram of lead and 1 kilogram of copper. What volume of water is necessary to fill the jar, the specific gravity of lead and copper being respectively 11.3 and 8.9?

11. Bought land at $60 an acre. How much must I ask an acre that I may deduct 25 % from my asking price, and yet make 20 % of the purchase price?

ADVANCED ARITHMETIC. — GUNTER BIBLE COLLEGE

1. (*a*) Define arithmetical complement, bank discount, an equation, specific gravity, tariff.

(*b*) Prove (do not merely illustrate) that to divide by a fraction one may multiply by the divisor inverted.

2. A man wishing to sell a horse and a cow asked three times as much for the horse as for the cow; but finding no purchaser, reduced the price of the horse 20 %, and the price of the cow 10 %, and sold both for $165. How much did he get for the cow?

3. How many acres are in a square the diagonal of which is 20 rods more than a side?

4. (*a*) Extract the sixth root of 1,073,741,824.

(*b*) Simplify $\frac{5 \div 5 + 5 \times 5 + 5 \div 5}{5 - 5 \div 5 + 5 \times 5 \div 5}$.

5. I sold a book at a loss of 25 %. Had it cost me $1 more, my loss would have been 40 %. Find its cost.

6. (*a*) Change 200332 in the quinary scale to an equivalent number in the decimal scale.

(*b*) Sum to infinity the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$.

7. If 100 grams of rock salt are dissolved in 1 liter of water without increasing its volume, what will be the specific gravity of the solution?

8. (*a*) If a ball of yarn 4 inches in diameter makes one pair of gloves, how many similar pairs will a ball 8 inches in diameter make?

(*b*) What must be paid for 6 % bonds to realize an income of 8 % on the investment?

9. Find the difference between the annual interest and compound interest of $ 6000 for 3 years 6 months at 10 %.

10. An article cost $ 6. At what price must it be marked so that the marked price may be reduced 22 % and still 30 % be gained?

11. At what two times between 3 and 4 o'clock are the hour and minute hands of a clock equally distant from 12?

FOR STATE CERTIFICATE. — NEW JERSEY

Commercial Arithmetic

1. Bought of Brown & Company the following bill of lumber: 8750 feet of boards at $ 31.33$\frac{1}{3}$ per M. feet; 5750 shingles at $ 5.25 per M.; 2860 laths at $ 2.87$\frac{1}{2}$ per M.; 520 joists, 20 feet long, 16 inches wide, and 3$\frac{1}{2}$ inches thick, at $ 15 per M. feet. Find the amount of the bill.

2. Find the sale price of a Brussels carpet 27 inches wide at $ 1.60 per yard for a room 15 feet long and 13$\frac{1}{2}$ feet wide if the strips run lengthwise.

3. Which will cost the more and how much, to lay a brick sidewalk 260 feet long and 4$\frac{1}{2}$ feet wide, estimating 8 bricks for each square foot of pavement at $ 12 per M., or to lay a flagstone walk at 22 cents per square foot?

4. How much will it cost to build two abutments for a bridge each 18 feet long at top and bottom, 12 feet wide at bottom and eight (8) feet wide at top and 11 feet high at $ 4.50 a perch for labor and stone?

5. Three men engaged in business. A furnished $ 6000 of capital; B $ 9600, and C $ 6400. They made a gain of $ 4800 and then sold out the business for $ 30,000. What was each one's share of gain?

6. What must I pay for a draft on Chicago for $ 475, payable 30 days after date, $\frac{1}{8}$ % premium, interest at 6 %?

ANSWERS AND SOLUTIONS

ARITHMETICAL PROBLEMS

1. (*a*) Let $\frac{2}{2}$ = distance the minute hand is ahead of the hour hand; $\frac{24}{2}$ = distance the minute hand moves while the hour hand travels $\frac{2}{2}$; $\frac{26}{2}$ = distance both travel = 120 spaces; $\frac{1}{2} = \frac{1}{26}$ of 120 spaces = $4\frac{8}{13}$ spaces; $\frac{2}{2}$ = 2 times $4\frac{8}{13}$ spaces = $9\frac{3}{13}$ spaces, the number of spaces the minute hand is in advance of the hour hand.

(*b*) Let $\frac{2}{2}$ = distance the hour hand has moved past 3 $\frac{24}{2}$ = distance the minute hand moved during the same time; $\frac{24}{2}$ = 15 minutes + $9\frac{3}{13}$ minutes + $\frac{2}{2}$; $\frac{22}{2} = 24\frac{3}{13}$ minutes; $\frac{1}{2} = \frac{1}{22}$ of $24\frac{3}{13}$ minutes = $\frac{315}{286}$ minutes; $\frac{24}{2}$ = 24 times $\frac{315}{286}$ minutes = $26\frac{62}{143}$ minutes, past 3.

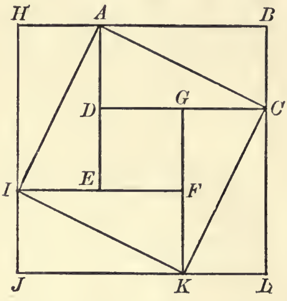
(*c*) Since the hands changed places, the minute hand fell short $9\frac{3}{13}$ minutes of going 2 hours. Therefore it was $26\frac{62}{143}$ minutes past 3 when I first looked, and 120 minutes − $9\frac{3}{13}$ minutes later = $17\frac{29}{143}$ minutes past 5, when I looked the second time, *Ans.*

2. The broken part of the tree, resting with the upper end on the ground and the other end attached to the stump, forms the hypotenuse of a right triangle, of which the base is 40 feet, and the altitude is the stump of the tree. The height of the tree may be found by the following rule, based on a demonstration in Geometry: From the square of the height subtract the square of the base, and divide the difference by twice the height. The height in this case is the height of the tree and not the height of the stump. Therefore $(120^2 - 40^2) \div (120 \times 2)$

$= 53\frac{1}{3}$, height of the stump. Then 120 feet $- 53\frac{1}{3}$ feet $= 66\frac{2}{3}$ feet, *Ans.*

3. The total number of dollars = 80 times the number of acres; or 20 times the number of acres = the number of dollars on one side of the boundary. One dollar is $1\frac{1}{2}$ inches in diameter; hence $\frac{3}{2}$ of 20 times, or 30 times, the number of acres = the number of inches on one side; $\frac{5}{2}$ times the number of acres = the number of feet on one side. Therefore ($\frac{5}{2}$ times the number of acres)$^2 \div 43{,}560$, or 5 times the square of the number of acres $\div 34{,}848$, = the number of acres; 5 times the square of the number of acres $= 34{,}848$ times the number of acres; or, $34{,}848 \div 5 = 6969.6$, the number of acres in the field, *Ans.*

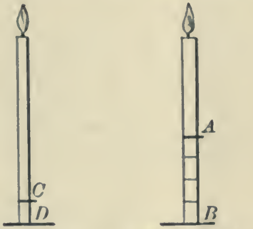
4. Let *ABCD* represent the rectangular field. Now suppose four such fields arranged in the form of a square by placing the short side of one against the long side of another, inclosing the square *DEFG*, as shown in figure.



Draw the diagonals *AC, CK, KI*, and *IA*. It may be readily shown that *ACKI* is a square; and since a diagonal is 100 rods, the area of the square *ACKI* = 10,000 square rods. One of the triangles, as *ACB*, has an area of 15 acres, or 2400 square rods. Hence the combined area of the 4 outer triangles $= 4 \times 2400 = 9600$ square rods. Adding this result to the area of the square *ACKI*, we have 19,600 square rods = the area of the square *HBLJ*. Hence, $BL = \sqrt{19{,}600} = 140$ rods.

Now from the area of the square *ACKI*, subtract the area of the four inner triangles, and we have the area of the square $DGFE = 400$ square rods. Hence $GF = \sqrt{400} = 20$ rods. Therefore, $BC = (140 - 20) \div 2 = 60$ rods, and $AB = 60 + 20 = 80$ rods, *Ans.*

5. Let A and C be the points the candles burn to, when No. 2 is 4 times No. 1. If CD be used as a unit of measure, AB will be equal to 4 such units.



Candle No. 1. Candle No. 2.

No. 1 burns in 4 hours.
No. 2 burns in 5 hours.

Now, if the candles be allowed to burn until CD is consumed, $\frac{4}{5}$ of a unit of AB will burn, leaving $3\frac{1}{5}$ units. Since the candles have been burning 4 hours, the $3\frac{1}{5}$ units remaining in No. 2 will be consumed in one hour if they continue to burn. Then $5 \times 3\frac{1}{5}$ units $= 16$ units, the number of units in each candle. Then since 16 units of No. 1 burn in 4 hours, 15 units of No. 1, or the part consumed, is burned in $\frac{15}{16}$ of 4 hours $= 3\frac{3}{4}$ hours, *Ans.*

6. The distance the lizard moves is the hypotenuse of a right triangle whose legs are 200 feet and 10 feet.

Ans. = 200.25 feet.

7.
$$\$1\left\{\begin{array}{r}\$3.50\\ 1.50\\ \\ .50\end{array}\right.\begin{array}{|c|c||c|c||c|c||l}\frac{2}{5} & & 2 & & 16 & & 16\text{ calves.}\\ & 2 & & 2 & & 2 & 2\text{ sheep.}\\ & & & & & & \\ 2 & 2 & 10 & 2 & 80 & 2 & \underline{82}\text{ lambs.}\end{array}$$
$$100\text{ number of head}$$

Another answer is 10, 20, and 70.

8. The daughter's share = daughter's share.
The wife's share = 2 daughter's share.
The son's share = 4 daughter's share.

D.'s + W.'s + S.'s = 7 times daughter's share.

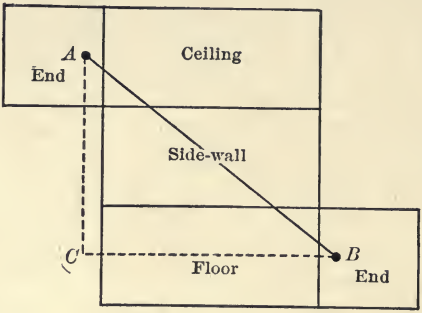
∴ 7 times daughter's share = the estate.

∴ daughter's share = $\frac{1}{7}$ of the estate,
wife's share = $\frac{2}{7}$ of the estate,
and son's share = $\frac{4}{7}$ of the estate.

9. 64.

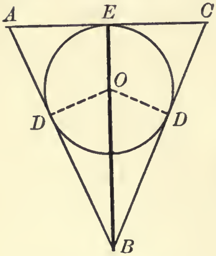
10. The distance AB is the hypotenuse of the right triangle

$$ABC = \sqrt{(32)^2 + (24)^2} = 40 \text{ feet.}$$



11. ∞ %.

12. First, let us find the volume of the largest ball that could be placed in the given cone and also the amount of water required to cover it.



Let AC in the diagram represent the diameter of the mouth of the glass, $BE =$ 4 inches, the altitude, and $OD =$ the radius of the largest marble which could be covered in the glass.

Area of $\triangle ABC =$ area of 3 △s of which OD is the altitude. Area of $\triangle ABC = 12$. Hence one half the radius of the largest marble $= 12 \div (5 + 5 + 6) = \frac{3}{4}$.

The diameter of the largest ball which could be covered in the glass $= 3$.

$$\therefore \text{ V of cone } ABC = \frac{1}{3}\pi r^2 h = \frac{36\,\pi}{3} = 12\,\pi.$$

$$\text{V of largest marble} = \frac{1}{6}\pi d^3 = \frac{9\,\pi}{2}.$$

$12\,\pi - \dfrac{9\,\pi}{2} =$ amount of water it takes to cover the largest marble. Now, the water which would cover the largest marble is to the water covering the required marble as the largest marble is to the required marble.

$$\therefore \left(12\,\pi - \frac{9\,\pi}{2}\right) : 4\,\pi :: 3^3 : x^3.$$

$\therefore x = 2.433$ inches, *Ans.*

13. The discount is $\frac{9}{100}$ of the face of the note. The interest is 10 % of the proceeds.

Hence, 10 % of the proceeds = 9 % of the face.

$\frac{1}{100}$ of the proceeds $= \frac{1}{10}$ of $\frac{9}{100} = \frac{9}{1000}$ of the face.

$\frac{100}{100}$ of the proceeds $= 100 \times \frac{9}{1000} = \frac{900}{1000}$ of the face.

$\frac{1000}{1000} - \frac{900}{1000} = \frac{100}{1000} = \frac{1}{10}$ of the face, which is the discount for the required time.

$\therefore$ the time is $\frac{1}{10} \div \frac{9}{100} = 1\frac{1}{9}$ years = 400 days.

14. Since it was a perfect power, the right-hand period must have been 25, and the last figure of the root must have been 5. Hence the last trial divisor was $1225 \div 5 = 245$. Then by the rule for extracting the square root, we know 5 to have been annexed to 24, which must have been double the root already found. That portion of the root, then, must have been $\frac{1}{2}$ of $24 = 12$. The entire root was 125. Therefore the power was $125^2 = 15{,}625$, *Ans.*

15. The length of the lawn is $\frac{3}{2}$ of its width, and if $\frac{1}{3}$ of it be taken off by a line parallel to the end, a square will be left, the side of which is the width of the lawn. The area of the lawn $= \frac{3}{2}$ the area of the square. If the dimensions of the lawn be increased 1 ft., its area will be equivalent to the area of $\frac{3}{2}$ of the square $+ \frac{3}{2}$ of a strip 1 foot wide $+ \frac{2}{2}$ of a strip 1 foot wide + a square with an area of 1 square foot = 651 square feet. Area of $\frac{3}{2}$ of the square $+ \frac{5}{2}$ of a strip 1 foot wide = 650 square feet. Taking $\frac{2}{3}$ of this quantity, we have the square $+ \frac{5}{3}$ of a strip 1 foot wide $= \frac{2}{3}$ of 650 square feet = 62,400 square inches. But $\frac{5}{3}$ of a strip 1 foot wide = a strip of the same length $\frac{5}{3}$ of a foot wide = two strips the same length $\frac{5}{6}$ of a foot wide, or 10 inches wide.

Now, if we place these two strips on adjacent sides of the square and also a square containing 100 square inches at the corner, we will have a new square the area of which = 62,400 square inches + 100 square inches = 62,500 square inches. A side of this square = 250 inches. Therefore the width of the lawn = 250 inches − 10 inches = 240 inches, or 20 feet, and the length = 30 ft., *Ans.*

16. Let 100 % = cost.
180 % = marked price.
140 % = selling price.
$\therefore \frac{180}{140} = 1\frac{2}{7}$ = length in yards.

17. The area of a triangle whose sides are 13, 14, and 15 feet may be found by the rule: "Add the three sides together and take half the sum; from the half sum subtract each side separately; multiply the half sum and the remainders together and extract the square root of the product."

13 feet + 14 feet + 15 feet = 42 feet = sum of sides.
$\frac{1}{2}$ of 42 feet = half sum of sides.
21 feet − 13 feet = 8 feet.
21 feet − 14 feet = 7 feet.
21 feet − 15 feet = 6 feet.

7056 = product of the half sum and the three remainders.

$\sqrt{7056} = 84$ square feet, area of triangle with sides 13, 14, and 15 feet.

Area of given triangle is 24,276 square feet.

The problem now becomes merely a comparison of areas, the larger triangle having sides in the same proportion as the smaller. Similar surfaces are to each other as the squares of their like dimensions, therefore, $84 : 24{,}276 :: 13^2$: the square of the corresponding side. Or, $\sqrt{24{,}276 \times 169 \div 84} = 221$, length of the corresponding side.

Similarly with 14 and 15 we find the other corresponding sides = 238 and 255. *Ans.* 221, 238, and 255.

18. Since my mistake was 55 minutes, the hands must have been 5 minute spaces apart. At 2 o'clock they were 10 spaces apart, hence the minute hand had gained 5 spaces. It gained 55 spaces in 1 hour, hence to gain 5 spaces requires $\frac{1}{11}$ of an hour, or $5\frac{5}{11}$ minutes. Therefore, it was $5\frac{5}{11}$ minutes past 2 o'clock, *Ans.*

19. The area of the whole slate = 108 square inches. The area of the frame = $\frac{1}{4}$ of 108 square inches = 27 square inches. Now, suppose 4 slates so placed as to form a square 9 + 12, or 21 inches, on a side. The whole area of this square = 441 square inches. 441 square inches − 4 × 27 square inches = 108 square inches, the area of the frames of the four slates = 338 square inches, the area of a square formed by the four slates without frames.

$\sqrt{338}$ square inches = 18.242 inches, a side of the square. Then since 21 inches includes 4 widths of the frame, 21 inches − 18.242 inches = 4 times the width of the frame.

Therefore the frame is .6895 inch wide.

20. 6 acres + 72* growths keep 16 oxen 12 weeks, or 1 ox for 192 weeks.

3 acres + 36 growths keep 16 oxen 6 weeks, or 1 ox for 96 weeks.

Adding the above, we have,

9 acres + 108 growths keep 16 oxen 18 weeks, or 1 ox for 288 weeks.

9 acres + 81 growths keeps 26 oxen 9 weeks, or 1 ox 234 weeks.

Subtracting, 27 growths keep 1 ox 288 weeks − 234 weeks, or 54 weeks.

∴ 1 growth keeps 1 ox 2 weeks.

∴ 150 growths keep 1 ox 300 weeks.

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* A growth is the weekly growth on one acre.

Also, 72 growths keep 1 ox 144 weeks.

Then, 6 acres keep 1 ox 192 weeks − 144 weeks, or 48 weeks.

6 acres keep 1 ox 48 weeks, 1 acre keeps 1 ox 8 weeks.

15 acres keep 1 ox 120 weeks.

∴ 15 acres + 150 growths keep 1 ox 120 weeks + 300 weeks, or 420 weeks.

Hence the number of oxen required is $420 \div 10 = 42$, *Ans.*

21. 400.

22. Precedence is given to the signs × and ÷ over the signs + and −; hence the operations of multiplication and division should always be performed before addition and subtraction. *Ans.* = 8.

23. ∞. **24.** 0. **25.** 2.236 minutes.

26. 20 feet. **27.** 28.44^+.

28. The distance from the extreme point of the given ball to the corner is to the distance of the nearest point of the given ball from the corner, as the diameter of the given ball is to the diameter of the required ball.

12 feet = an edge of the cube. Then $\sqrt{3 \times 144} = 20.7846$, the distance from a lower to the opposite upper corner of the room. $20.7846 - 12 =$ twice the distance from the given ball to the corner. $4.3923 =$ the distance of the nearest point of the given ball from the corner. Then, the distance from the extreme point of the ball to the corner $= 20.7846 - 4.3923 = 16.3923$.

∴ 16.3923 feet : 4.3923 feet : : 12 feet : (3.215 feet), *Ans.*

29. Let $\frac{2}{2} =$ number of minutes past 3 o'clock.

$40 - \frac{2}{2} =$ distance the minute hand is from 8.

$\frac{2}{24} =$ number of minute spaces the hour hand is from 3.

$15 + \frac{2}{24} =$ distance the hour hand is from 12. But since the minute hand is the same distance from 8 that the hour hand is from 12, then

$$40 - \frac{24}{24} = 15 + \frac{2}{24}.$$

$$\therefore \frac{2}{2}, \text{ or } \frac{24}{24} = 23\tfrac{1}{13} \text{ minute past 3 o'clock, } \textit{Ans.}$$

30. Since an edge of the given cube differs from an edge of the original cube by 2 inches, the difference in the solidity of the cubes will be the solidity of 7 blocks 2 inches thick — a corner cube, 3 narrow blocks, and 3 square blocks. The contents of these 7 solids = 39,368 cubic inches. By taking away the 8 cubic inches, the number of cubic inches in the corner cube, there remains 39,360 cubic inches, the solidity of the 3 narrow blocks and 3 square blocks. Then 1 square block and 1 narrow block contain 13,120 cubic inches.

Now, since these blocks are 2 inches in thickness, the sum of the areas of 1 face in each of the 2 = 6560 square inches. That is, the area of a square and a rectangle 2 inches in width = 6560 square inches. This rectangle is equivalent to 2 rectangles of equal length and 1 inch wide. Now, if we place these rectangles on adjacent sides of the square and also add a square 1 square inch in area to complete the square, we will have a square = 6561 square inches. A side of this square $= \sqrt{6561} = 81$ inches = an edge of the original cube after the reduction, increased by 1 inch.

$\therefore$ an edge of the original cube = 82 inches, *Ans.*

31. The required number is the remainder left after subtracting the largest cube. In extracting the cube root of 592,788 we find 84 to be a side of the largest cube, and 84 to be the remainder. $\therefore$ 84 is the required number, *Ans.*

32. 80; 40.

33. In 1 hour A can row upstream $\frac{1}{3}$ of the distance. In 1 hour A can row downstream $\frac{1}{2}$ of the distance. $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$, or twice the distance the stream flows in 1 hour. Hence, the stream flows $\frac{1}{12}$ of the distance in 1 hour. $\frac{1}{12}$ of the distance = 1 mile. $\therefore$ the distance = 12 miles, *Ans.*

34. B's share $= \frac{1}{5}(90 + 20) = 22$; $22 - 20 = 2$, the loss.

35. The required number is the remainder left after subtracting the largest square. In extracting the square root of

13,340 we find 115 to be the whole number of the root and 115 the remainder. $\therefore$ 115 is the required number, *Ans.*

36. $\sqrt[3]{\text{number}} = 10\sqrt[4]{\text{number}}$. Raising to the 12th power, $(\text{number})^4 = 10^{12}\,(\text{number})^3$. Dividing by $(\text{number})^3$, we have the number $= 10^{12} = 1{,}000{,}000{,}000{,}000$.

37. $\frac{1}{10}$. **38.** 0. **39.** $66\frac{2}{3}\,\%$.

40. This problem may be solved by Geometrical Progression. $l = ar^{n-1}$. $l = \frac{343}{512}$, the last term. $a = 1$, the first term. $n = 4$, the number of terms.

$$\therefore \tfrac{343}{512} = r^3, \text{ and } r = \tfrac{7}{8}.$$
$$\therefore 1 - \tfrac{7}{8} = \tfrac{1}{8}, \text{ or } 12\tfrac{1}{2}\,\%.$$

41. \$200; \$12. **42.** 30. **43.** 20 %. **44.** \$750.

45. \$37.037. **46.** 10:40 o'clock. **47.** $3\frac{1}{13}$ cords.

48. Let $100\,\% =$ cost of goods.
$180\,\% =$ marked price of goods.
$\frac{1}{6}$ of $180\,\% = 30\,\%$, loss.
$180\,\% - 30\,\% = 150\,\% =$ selling price.
$150\,\% - 100\,\% = 50\,\%$, gain, *Ans.*

49. $\sqrt[3]{1} : \sqrt[3]{\frac{7}{8}} :: 6 : (5.738)$, the diameter of the inside sphere. 6 inches $-$ 5.738 inches $=$ .262 inch, twice the thickness of the shell. $\therefore$ .131 inch $=$ the thickness of the shell.

50. The number of bushels of apples $= \frac{4}{5}$ of 20 bushels $=$ 16 bushels.

51. Let $100\,\% =$ present worth of sales.
$103\,\%$ of present worth of sales $= 95\,\%$ of sales.
$1\,\%$ of present worth of sales $= \frac{95}{103}\,\%$ of sales.
$100\,\%$ of present worth of sales $= 92\frac{24}{103}\,\%$ of sales.
$\therefore 92\frac{24}{103}\,\%$ of sales $= 119\frac{1}{2}\,\%$ of cost of goods.
$1\,\%$ of sales $= 1.29\frac{107}{190}\,\%$ of cost of goods.
$100\,\%$ of sales $= 129\frac{107}{190}\,\%$ of cost of goods.
$\therefore 29\frac{107}{190}\,\% =$ the per cent advance of the cost.

52. $84{,}245{,}000 - 48{,}245{,}000 = 36{,}000{,}000$.
$36{,}000{,}000 \div 36{,}000 = 1000$, the divisor, *Ans.*

53. 1.754 inches; 2.246 inches; 4 inches.

54. $3\frac{3}{4}$ years.

55. $300.

56. 300 miles.

57. 60 days; 40 days.

58. $1600 \div 80 = 20$, the difference of the two numbers. The sum + the difference = twice the greater number. Hence, $80 + 20 = 100$ = twice the greater number.
∴ 50 = the greater number, and 30 = the smaller number.

59. 50%.

60. 15.

61. 3.

62.

$$30\left\{\begin{array}{c|c||c|c|} 0 & \frac{1}{80} & 2 & 10 \\ 90 & \frac{1}{60} & 1 & 5 \end{array}\right.$$

10 gallons of water, *Ans.*

63. Solve by means of Progression:

Let P = principle; r = rate of interest; n = number of years; A = amount of each payment. Then

$$A = \frac{r \cdot P(1+r)^n}{(1+r)^n - 1}.$$

Since one amount is paid at the beginning of the year, the principal less that amount will be the money to reckon as the new principal for the term of 4 years.

$$\$1000 - A = (P - A).$$

$$A = \frac{r(P-A)(1+r)^n}{(1+r)^n - 1} = \frac{\frac{1}{10}(P-A)(1+\frac{1}{10})^4}{(1+\frac{1}{10})^4 - 1}$$

$$= \frac{\frac{1}{10}(\$1000 - A)(1.4641)}{.4641}.$$

Clearing of fractions,

$$\$.4641\,A = \tfrac{1}{10}(\$1464.1 - 1.4641\,A).$$
$$6.1051\,A = \$146.41.$$

$$\therefore A = \$239.81, \quad \textit{Ans.}$$

64. $3\frac{21}{68}$%.

65. $$.67\frac{1}{2}$.

66. The true discount on \$1 is $\$1-(\$1\div 1.015)=\$.014\frac{790}{1015}$.
The bank discount on \$1 is \$.015.
Then $\$.015-\$.014\frac{790}{1015}=.000\frac{225}{1015}$, the difference.
$\$.90\div .000\frac{225}{1015}=\$4060$, the face of the note, *Ans.*

67. \$50.

68. 8 days.

69. $66\frac{2}{3}\%$.

70. 6 feet.

71. 168.298^{+} bushels.

72. 8 pounds.

73. $11\frac{1}{4}$ ounces.

74. $3\frac{1}{2}$ years.

75. \$160.

76. 25 dozen; 92 cents.

77. 62.832 minutes.

78. $5\frac{5}{31}$ hours.

79. 37 inches.

80. \$76.52, first; \$96.52, second.

81. 30 steps. **82** 104 feet.

83. $27\frac{3}{11}$ minutes after 5 o'clock.

84. $10\frac{10}{11}$ minutes past 2 o'clock.

85. A pound of feathers.

86. 600; 1200; 1800; 2400 yards.

87. 360 acres. **88.** \$20. **89.** 1,000,000. **90.** \$2.

91. Let $100\% =$ the marked price.
He receives $100\%-10\%=90\%$.

Since he uses a yard measure .72 of an inch too short, he gives only $35\frac{7}{25}$ inches for 1 yard. He sells $35\frac{7}{25}$ inches for 90% of the marked price. Therefore he would sell 36 inches for $91\frac{41}{49}\%$ of the marked price.

$\therefore 100\%-91\frac{41}{49}\%=8\frac{8}{49}\%$, the required discount.

92. 69.36286^{+} pounds.

93. 32 feet^{+}.

94. 8%.

95. \$8.75.

96. Book, \$1.10; pen, \$.10.

97. 17%.

98. Each new day begins at the 180th meridian, which was crossed in the Pacific Ocean before reaching Manila.

99. 7 sheep.

100. $\frac{2}{9}$.

101. 40.

102. 72.

103. Friday.

104. 0.

105. 3 P.M.

106. A, $500; B, $700.

107. 6:40 P.M.

108. B paid $92; 15% gain.

109.

$$\begin{aligned}\text{The greater} + \text{the less} &= 582.\\ \text{The greater} - \text{the less} &= 218.\\ \therefore\ 2 \text{ times the less} &= 364,\\ \text{and the less} &= 182.\\ \text{The greater} &= 400.\end{aligned}$$

110. 2760.4288⁺ cubic inches; 1152 square inches.

111. A's, $90; B's, $135; C's, $180.

112. $20.

113. August 11 was 21 days before the note was due. The use of any sum of money for 21 days, or $\frac{7}{10}$ of a month, at 6% is equal to $\frac{1}{100}$ of it. Then, since he promised to pay such a sum that the use of it for 21 days was to equal the use of the sum unpaid for 2 months, $\frac{1}{100}$ of the sum unpaid $= \frac{7}{2000}$ of the sum paid. Hence the sum unpaid $= \frac{700}{2000}$ of the sum paid. $\therefore \frac{2000}{2000}$ of the sum paid $+ \frac{700}{2000}$ of the sum paid $=$ $100. $\therefore$ the sum paid $=$ $74.07.

114. $212.12.

115. $246.60.

116. $50 gain.

117. 43.817 pounds.

118. 64 pounds.

119. 7 cents to A; 1 cent to B

120. 10.

121. 45 feet.

122. 30 of first quality; 16 of second quality.

123. 32 miles.

124. $2; $1.

125. 810 revolutions.

126. 16 dozen.

127. A, 2.87 rods; B, 4.72 rods; C, 13.82 rods.

128. 1,000,000.

129. Horse, \$110; cow, \$10.

130. 216 pounds.

131. \$850.

132. \$4.

133. They are the same.

134. 2.

135. 20.

136. 60.

137. 20%.

138. 20%.

139. First, \$250; second, \$200.

140. Husband's age, 24 years; wife's age, 20 years.

141. 20 gallons of wine; 30 gallons of water.

142. 1300. **143.** 4. **144.** \$.80. **145.** \$.75. **146.** 8.

147. $21\frac{9}{11}$ minutes past 4 o'clock.

148. $10\frac{10}{11}$ minutes past 2 o'clock.

149. $27\frac{3}{11}$ minutes past 2 o'clock; 3 o'clock.

150. $43\frac{7}{11}$ minutes past 2 o'clock.

151. .50.

152. 245.574.

153. Wife, \$8500; son, \$12,750; daughter, \$2125.

154. 1 mile.

155. 180.

156. 43,200.

157. 1,860,867.

158. $9\frac{23}{33}\%$, or 9.69^+.

159. \$42,949,672.95.

160. \$4.

161. Midnight.

162. 1 hour and 20 minutes is lost in going 50 miles.
∴ 80 minutes is lost in going 50 miles.
∴ 1 minute is lost in going $\frac{5}{8}$ mile.
∴ 120 minutes is lost in going 75 miles.
∴ 2 hours is lost in going 75 miles.
But 2 hours is the entire time lost.
∴ the distance traveled after the breakdown is 75 miles.

Again, the train at its original speed goes as far in 3 hours as it went in 5 hours at its speed after the breakdown,

∴ in 3 hours at the original speed it goes 75 miles.

∴ in 1 hour at the original speed it goes 25 miles.

∴ the length of the line is 75 miles + 25 miles = 100 miles.

163. $11\frac{1}{4}$ cents. **166.** $1\frac{1}{2}$. **168.** 300 feet.

165. 2. **167.** $4\frac{4}{9}$. **169.** 2 : 1.

170. 2 miles 340 feet.

171. 132 and 140. **172.** 20 %. **173.** By their sum.

174. James's speed $= \frac{96}{100}$ of my speed.

John's speed $= \frac{190}{200}$ of James's speed.

∴ James's speed $= \frac{190}{200}$ of ($\frac{96}{100}$ of my speed).

∴ James's speed $= \frac{114}{125}$, or $\frac{456}{500}$ of my speed.

∴ James's speed and my speed are in the ratio of 456 to 500.

∴ in running 500 yards I beat James 500 yards − 456 yards = 44 yards, *Ans.*

175. First, .759 inch; second, 1.08^{+} inches;. third, 4.16^{+} inches.

176. *First Method.* 1. Any remainder which exactly divides the previous divisor is a common divisor of the two given quantities.

2. The greatest common divisor will divide each remainder, and cannot be greater than any remainder.

3. Therefore, any remainder which exactly divides the previous divisor is the greatest common divisor.

Second Method. 1. Each remainder is a number of times the greatest common divisor. For a number of times the greatest common divisor, subtracted from another number of times the greatest common divisor, leaves a number of times the greatest common divisor.

2. A remainder cannot exactly divide the previous divisor unless such remainder is once the greatest common divisor.

3. Hence, the remainder which exactly divides the previous divisor, is once the greatest common divisor.

177. 112 cubic feet.

178. In 5 seconds both trains travel 600 feet.

∴ in 1 hour both trains travel $81\frac{9}{11}$ miles.

In 15 seconds the faster train gains 600 feet.

∴ in 1 hour the faster train gains $27\frac{3}{11}$ miles.

Now, we have the sum of their rates $= 81\frac{9}{11}$ miles and the difference of their rates $= 27\frac{3}{11}$ miles.

∴ rate of faster + rate of slower $= 81\frac{9}{11}$ miles, and rate of faster − rate of slower $= 27\frac{3}{11}$ miles.

∴ 2 times rate of faster $= 109\frac{1}{11}$ miles.

∴ rate of faster $= 54\frac{6}{11}$ miles.

Also, 2 times rate of slower $= 54\frac{6}{11}$ miles.

∴ rate of slower $= 27\frac{3}{11}$ miles.

179. 1.118 times.

180. $38\,t\,e\,6$.

181. Senary.

182. 1,110,100,010 years.

183. 22144_6.

184. 10212_5.

185. 30423151_7.

186. 3424_5.

187. 13_9.

188. 12_4.

189. 128.

190. 180.

191. 658,548,918.

192. 28 gallons wine; 42 gallons water.

193. $10\frac{5}{7}$.

194. Since the numbers are consecutive, each must lie near the cube root of 15,600; in other words, the numbers must lie between 20 and 30. Now, 15,600 is divisible by 25, since it ends in two ciphers, hence 25 may be one of the numbers. By trial, we find that 624 would be the product of the other two, which themselves must end in 4 and 6 to give a product ending in 4. *Ans.* 24; 25; 26.

195. Such a number must lie halfway between 1042 and 1236.

$\therefore 1236 - 1042 = 194$, which divided by 2, gives 97.

$\therefore 1042 + 97 = 1139$, *Ans.*

196. 76,809,256,566.

197. 49. The remainder left over after subtracting the largest cube is the number.

198. At 4 miles per hour = 1 mile in 15 minutes, and 5 miles per hour = 1 mile in 12 minutes.

$\therefore$ in going 1 mile there is a difference of 3 minutes, but the actual difference is 10 minutes + 5 minutes = 15 minutes.

$\therefore$ 15 minutes ÷ 3 minutes = 5. *Ans.* 5 miles.

199. $\frac{1}{2}$ of small glass = $\frac{1}{6}$ of total, and since the large glass is $\frac{2}{3}$ of both, $\frac{1}{3}$ of the large glass = $\frac{2}{9}$ of total, and $\frac{1}{6} + \frac{2}{9} = \frac{7}{18}$ = wine.

$\therefore \frac{11}{18}$ = water. — From "Arithmetical Wrinkles."

200. When the ball just floats, its specific gravity is 1. Then by Allegation, we have

$$1 \left\{ \begin{array}{c|c|c} \frac{1}{36} & \frac{35}{36} & 324 \\ 10 & 9 & 35 \end{array} \right\}, \text{ or the lead ball is } \frac{35}{359} \text{ of the globe.}$$

$\frac{35}{359}$ of $\frac{4}{3}\pi(12)^3 = \frac{80640}{359}\pi$, and $\sqrt[3]{\left(\frac{80640}{359}\pi \div \frac{4}{3}\pi\right)} = 5.52$ inches, radius of ball, and $12 - 5.52$, or 6.48, inches is the thickness of the shell. — From "The School Visitor."

201. 14° F. = – 10° C. and 270° F. = $132\frac{2}{9}$° C. The specific heat of ice is .505, that of steam is .48, latent heat of fusion is 80, and that of evaporation is 537; then,

$$100(10 \times .505 + 80 + 100) = 18{,}505 \text{ heat units,}$$

required to melt the ice and raise its temperature to 100° C.

There are $80 \times 32\frac{2}{9} \times .48 = 1237\frac{1}{3}$ heat units given off in reducing the steam at $132\frac{2}{9}$° C. to steam at 100° C.

There are $(18{,}505 - 1237\frac{1}{3}) \div 537 = 32.16$ pounds of steam at 100° C. to be condensed to water at 100° C. The result would be 132.16 pounds of water at 100° C., and $80 - 32.16 = 47.84$ pounds of steam at 100° C.

— From "The School Visitor."

202. If the average for the entire distance were 30 miles an hour, 50×30 or 1500 miles would be run, but this lacks 300 miles which must be made up running 55 miles per hour, or 25 miles an hour faster, taking $300 \div 25$, or 12 hours. Hence, the distance from B to C is 12×55, or 660 miles, and $(50 - 12)$ times 30, gives 1140 miles from A to B.

— From "The School Visitor."

203. Volume of sphere = 2 times volume of double cone. Surface of sphere = $\sqrt{2}$ times surface of double cone.

204. 20 rods.

205. For bodies above the earth's surface, the weight varies inversely as the square of the distances from the center. Hence, to weigh $\frac{1}{16}$ as much as at the surface, the body must be $\sqrt{16} = 4$ times as far from the center, or 16,000 miles, and the required height above the surface is $16{,}000 - 4000 = 12{,}000$ miles.

206. 1 mile. **207.** 7.2 inches. **208.** 34.

209. The difference between the bank and the true discount is always the interest on the true discount. Hence \$9 is 12% of the true discount, which is \$75. The bank discount is \$9 more, or \$84, which is 12% of the face of the note, and then \$84 divided by .12 gives \$700, the face.

210. By the prismoidal formula, the volume V is $\frac{1}{6}$ of (upper base + lower base + 4 times middle section) × length. Therefore $V = \frac{1}{6}\,(4 \times 4 + 2 \times 3 + 4 \times 3 \times 3\frac{1}{2}) \times 120 \div 144 = 8\frac{8}{9}$ feet, *Ans.*

211. Place the box on its end and put in 11 rows of 5 and 4 balls, alternately, making a total of 50 balls in the first layer.

Place the second layer in the hollows of the first, and it has 6 rows of 4 each and 5 rows of 5 each, making 49 balls in the second layer. In this manner 12 layers may be placed, making a total of $(50 + 49) \times 6 = 594$ balls.

— From "The Ohio Teacher."

212. If the field were 48 feet wide, it would take one post less at each end and two less at each side, or 6 less; but to make 66 less, the field must be $11 \times 48 = 528$ feet, or 32 rods wide, and 64 rods long; area, 12.8 acres.

213. 17.584, specific gravity.

214. $7\frac{1}{2}$ feet, the distance the ball bounds. 30 feet equals the whole distance the ball moves.

215. Let $r =$ rate per month, $12r =$ rate per annum, $p =$ sum borrowed, $n =$ number of payments, $q =$ cash payment. Then, from algebra, we get

$$q = \frac{pr(1+r)^n}{(1+r)^n - 1}, \ q = 9\tfrac{1}{2}, \ p = \$500, \ n = 72.$$

$$\therefore \ (q - pr)(1 - r)^n = q, \text{ and } (19 - 1000\,r)(1 + r)^{72} = 19.$$

$$\therefore \ r = .00911, \text{ and } 12\,r = .10932 = 10.932\,\%.$$

216. 1178.1 square feet.

217. $5\frac{201}{468}$ ounces.

218. 6.864^+ inches.

219. 72 and 96.

220. Since the numbers have a common factor plus the same remainder, if the numbers are subtracted from one another, the results will contain the common factor without the remainder, thus:

$$\begin{array}{rrr} 364 & 414 & 539 \\ & 364 & 414 \\ \hline & 50 & 125 \end{array}$$

The largest number that will divide all of these numbers is 25, *Ans.*

221. I. Let $S =$ selling price and $C =$ cost.

Then, $S - C =$ gain and $\frac{S - C}{C} =$ rate of gain.

Also, $S - \frac{92}{100}C =$ supposed gain, and

$$\frac{S - \frac{92}{100}C}{\frac{92}{100}C} = \frac{\frac{100}{92}S - C}{C} = \text{supposed rate of gain.}$$

$$\therefore \frac{\frac{100}{92}S - C}{C} - \frac{S - C}{C} = \frac{1}{10}, \text{ or } 10\%.$$

$$\frac{80}{92}S = C, \text{ or } S = \frac{92}{80}C.$$

$$\therefore S - C = \frac{12}{80}C, \text{ or } 15\%, \textit{ Ans.}$$

II. *A Short Solution.* $8 : 10 = 92 : 115.$

$115 - 100 = 15\%$ gain.

222. Let $\frac{2}{2} =$ distance the hour hand moves past 3 o'clock.
$\frac{24}{2} =$ distance the minute hand moves in the same time.
Then $\frac{24}{2} + \frac{2}{2} = \frac{26}{2} =$ distance they both move.
But the distance they both move $= 45$ minutes.

$\therefore \frac{26}{2} = 45$ minutes.

$\frac{1}{2} = \frac{1}{26}$ of 45 minutes $= 1\frac{19}{26}$ minutes.

$\frac{24}{2} = 24 \times 1\frac{19}{26}$ minutes $= 41\frac{7}{13}$ minutes.

$\therefore$ It is $41\frac{7}{13}$ minutes past 3 o'clock.

223. The weight of the first ball is $3\frac{3}{8}$ times an equal bulk of water, and that of the second is $2\frac{10}{27}$ times the equal bulk of water; hence, $3\frac{3}{8}$ times the volume of the first equals $2\frac{10}{27}$ times the volume of the second ball. But the volumes vary as the cubes of the diameters; hence, the required diameter is,

$$d = \sqrt[3]{(3\tfrac{3}{8} \div 2\tfrac{10}{27})} = 1\tfrac{1}{8} \text{ feet, } \textit{Ans.}$$

224. The amount of \$500 for 2 years at 6% is \$560; \$2500 − \$560, or \$1940, is the amount of the note, the present worth of which, for 24 − 8, or 16 months, is \$1796.30.

225. The present worth of \$201 for 30 days at 6% is \$200; the present worth of \$224.40 for 4 months at 6% is \$220. Hence, the present rate of gain is (220 − 200) ÷ \$200 = 10%, *Ans.*

226. If the 65 minutes be counted on the face of the same clock, then the problem would be impossible, for the hands must coincide every $65\frac{5}{11}$ minutes as shown by its face, and it matters not if it runs fast or slow; but if it is measured by true time it gains $\frac{5}{11}$ of a minute in 65 minutes, or $\frac{60}{143}$ of a minute per hour, *Ans.*

227. The loss of weight of an immersed body equals the weight of the fluid displaced. Hence $970 - 892 = 78$ ounces, weight of water displaced, and $970 - 910 = 60$ ounces weight of alcohol displaced. But as water is taken as the standard of comparison, the specific gravity of alcohol is $60 \div 78 = \frac{10}{13} = .769^{+}$, *Ans.*

228. The rate of the ship is $\frac{1}{3}^\circ$ per hour, while that of the sun is 15°. When they both move west, the sun gains $14\frac{2}{3}^\circ$; but when the ship moves east the sun gains $15\frac{1}{3}^\circ$. Therefore since the sun must make a gain of 360° in each case, the time from noon to noon is $360^\circ \div 14\frac{2}{3} = 24\frac{6}{11}$ hours, west and $360^\circ \div 15\frac{1}{3}^\circ = 23\frac{11}{23}$ hours east.

229. $\frac{1}{3}$ of 165 acres = 55 acres, the amount of land each man should furnish.

100 acres − 55 acres = 45 acres, the number of acres A furnishes C.

65 acres − 55 acres = 10 acres, the number of acres B furnishes C.

Hence, $\frac{45}{55}$ of \$110 = \$90, the amount A should receive, and $\frac{10}{55}$ of \$110 = \$20, the amount B should receive.

230. Let r be the internal radius of the cup; and the volume of a quart of wine, $57\frac{3}{4}$ inches. Then $240\,\pi r^3 \div (3 \times 57\frac{3}{4}) =$ value of wine in cents.

Also $40\,\pi r^2 =$ value of cup in cents.

$$\therefore 40\,\pi r^2 = \frac{240\,\pi r^3}{3 \times 57\frac{3}{4}}.$$

$$\therefore r = 28\tfrac{7}{8} \text{ in.} \quad \therefore 40\,\pi r^2 = \$\,1047.74, \textit{ Ans.}$$

231. 11 times.

232. Eleven integral solutions, as follows:

Average price = 1	5	1	2	3	4	5	6	7	8	9	10	11
	1	91	82	73	64	55	46	37	28	19	10	1
	$\frac{1}{2}$	8	16	24	32	40	48	56	64	72	80	88

233. The volume is found by the prismoidal formula.

$\frac{1}{6}l(2 \times 2 + 1 \times 12 + 4 \times \frac{3}{2} \times \frac{14}{2}) \div 144 = \frac{29}{432}l$ feet, or if l be the length in feet, the board measure is $\frac{29}{36}$ of the length in feet.

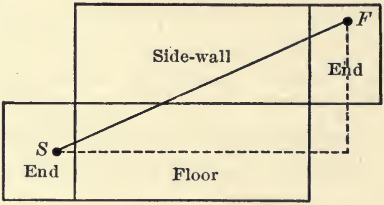
234. $1\frac{11}{18}$ board feet.

235. Since $.0\frac{1}{2} = .05$, $.\frac{1}{2}$ must be .5.

236. 96 acres.

237. 40 rods; 30 rods; $7\frac{1}{2}$ acres.

238. A liter of ice weighs 918 grams and a liter of sea water weighs 1030 grams. Then 918 divided by 1.03 equals 891.262 cubic centimeters displaced by one liter of ice, and 1000 − 891.262 is 108.738 cubic centimeters above water. Now 108.738 divided by 1000 gives .108738 of the whole above water, and 700 divided by .108738 equals 6437.5 cubic yards, the volume of the iceberg.



239. The distance SF is the hypotenuse of a right triangle $= \sqrt{(15)^2 + (39)^2} = 41.78^+$ ft.

240. \$563.23 due A.

241. (*a*) The least time required is $59\frac{13}{73}$ seconds past 12.

(*b*) The least time required is $30\frac{300}{1427}$ seconds past 12.

(*c*) The least time required is $1\frac{23}{697}$ minutes past 12.

242. \$2500.00.

243. \$225 = first payment; \$675 = second payment.

244. First, \$8400; second, \$7800; third, \$7280.

245. 8 yards of first kind; 16 yards of second kind.

246. $21\frac{9}{11}$ minutes past 4 o'clock.

247. A, $261\frac{3}{17}$ days; B, 120 days.

248. 10.

249. $7\frac{1}{9}$ feet; $8\frac{8}{9}$ feet.

250. 13,066.4 miles.

251. 2 inches.

252. 2 cones.

ALGEBRAIC PROBLEMS

1. Let $x =$ your age.
$y =$ difference between our ages.

Then $x + y =$ my age.

$$\therefore \frac{x+y}{2} + y = x,$$

and $$(x+y) + (x+2y) = 100.$$

Solving, $x = 33\frac{1}{3}$ and $x + y = 44\frac{4}{9}$.

2. A, 72 hours; B, 90 hours.

3. Let the time be x minutes past 10 o'clock. We assume that at the beginning of every minute the second hand points at 12 on the dial. The distance of the second hand from the minute hand at the required time is $60x - x = 59x$; and that of the second hand from the hour hand is

$$60 - 60x - (10 + \tfrac{1}{12}x) = 50 - 60x - \tfrac{1}{12}x.$$

$$\therefore 59x = 51 - 60x + \tfrac{1}{12}x.$$

Solving, $x = \frac{612}{1427}$ minutes $= 25\frac{1045}{1427}$ seconds.

— From "American Mathematical Monthly."

4. Let $s =$ distance between cars going in the same direction.

Let $t =$ interval of time between cars going in the same direction.

Let $x =$ rate of car.

Let $y =$ rate of man.

Then, $x - y =$ rate of approach when both travel in the same direction.

$x + y =$ rate of approach when they travel in opposite directions.

By conditions of problem,

$$12(x-y) = s = 4(x+y).$$

$$\therefore x = 2y.$$

Also, $$t = \frac{s}{x} = \frac{4(x+y)}{x} = 6.$$

Therefore the interval between cars is 6 minutes, and my rate is half the speed of the cars.

— From "School Science and Mathematics."

5. Let $x =$ the number of eggs for a shilling.

Then $\frac{1}{x} =$ the cost of one egg in shillings.

and $\frac{12}{x} =$ the cost of one dozen in shillings.

But if $x - 2 =$ the number of eggs for a shilling, then $\frac{12}{x-2}$ would be the cost of one dozen in shillings.

$$\therefore \frac{12}{x-2} - \frac{12}{x} = \frac{1}{12} \text{ (1 penny being } \tfrac{1}{12} \text{ of a shilling).}$$

Solving, $x = 18$ or -16. Then if 18 eggs cost a shilling, 1 dozen will cost $\frac{12}{18}$ of a shilling, or 8 pence, *Ans.*

6. Let $x =$ amount per yard received by one.

Then $x + \frac{1}{2} =$ amount per yard received by the other.

$$\therefore \frac{100}{x} + \frac{100}{x + \frac{1}{2}} = 100.$$

Solving, $x = 1.7808.$

One builds 56.15 yards at \$1.7808; other builds 43.85 yards at \$2.2808, *Ans.*

7. 1760 yards, or 1 mile.

8. Let $x =$ number of acres.

$160x =$ number of dollars for which the land sold.

Then since $1\frac{1}{2}$ inches = diameter of a dollar,

$1\frac{1}{2}(160x) = 240x =$ perimeter of square in inches.

$\frac{240x}{4}$, or $60x =$ length of one side of the square in inches.

$\frac{60x}{12}$, or $5x =$ length of one side of the square in feet.

$(5x)^2 =$ the number of square feet in the square.

$$\frac{25x^2}{43560} = \text{number of acres.}$$

$$\therefore \frac{25x^2}{43560} = x.$$

Solving, $x = 1742.4$, *Ans.*

9. 2652.5^+ feet; 2627.4^+ feet.

10. Let $x =$ one side of the square in feet.

Then $\frac{x^2}{43560} =$ the number of acres.

$16x =$ the number of feet of boards in the fence.

$\frac{16x}{11} =$ the number of boards in the fence.

$$\therefore \frac{x^2}{43560} = \frac{16x}{11}.$$

Solving, $x = 63{,}360.$

Then $\frac{x^2}{43560} = 92{,}160$ acres, or 144 sections.

11. $10\frac{5}{41}$ hours.

12. Let $x =$ rate of faster train per hour in miles.
$y =$ rate of slower train per hour in miles.
In 5 seconds both trains travel 600 feet.
$\therefore$ in one hour they travel $81\frac{9}{11}$ miles, or

$$x + y = 81\tfrac{9}{11}. \qquad (1)$$

In seconds the fast train gains 600 feet.
$\therefore$ in one hour the fast train gains $27\frac{3}{11}$ miles, or

$$x - y = 27\tfrac{3}{11}. \qquad (2)$$

Solving, $x = 54\frac{6}{11}$, and $y = 27\frac{3}{11}$, *Ans.*

13. Let $x =$ number of minutes until 6 o'clock.
Then 6 hours $- x =$ time past noon.
3 hours $+ 4x =$ time past noon 50 minutes ago.

$$\therefore 360 - x = 180 + 4x + 50.$$

Solving, $x = 26$, *Ans.*

14. Let $x =$ cost of the gun in dollars.

$$\frac{x}{100} = \text{per cent of loss.}$$

Then

$$\left(\frac{x}{100}\right)x = \text{loss.}$$

$$\therefore x - \frac{x^2}{100} = 9.$$

Solving, $x = 90$, or 10.

$\therefore$ \$90, or \$10 = the cost of the gun.

15. Let $x =$ number of eggs he brought.
Then $x + 1 = \frac{2}{3}$ of them,
and $\frac{3}{2}(x + 1) =$ number of eggs in the nest.
Also $x - 2 = \frac{1}{2}$ of them,
and $2(x - 2) =$ number of eggs in the nest.

$$\therefore 2(x - 2) = \tfrac{3}{2}(x + 1).$$

Solving, $x = 11$.
Then $2(x - 2) = 18$, *Ans.*

16. Let $x =$ cost of lot in dollars.

$$\frac{x}{100} = \text{per cent of gain.}$$

Then $$\frac{x^2}{100} = \text{gain.}$$

$$\therefore x + \frac{x^2}{100} = 144.$$

Solving, $x = 80$, or -180. $\$80 =$ *Ans.*

17. 6 inches. 18. $\sqrt[4]{3}$. 19. $\frac{3}{2} \pm \frac{1}{2}\sqrt{5}$ and $\frac{1}{2} \pm \frac{1}{2}\sqrt{5}$.

20. 21 minutes $49\frac{1}{11}$ seconds past 4 o'clock.

21. Let $x =$ number of men in a side of the first square.

Then $x^2 =$ number of men in the first square,

and $x^2 + 39 =$ number of men.

Also $x + 1 =$ number of men in a side of the second square

Then $(x+1)^2 =$ number of men in the second square,

and $(x+1)^2 - 50 =$ number of men.

$\therefore (x+1)^2 - 50 = x^2 + 39.$

Solving, $x = 44.$

Then $x^2 + 39 = 1975$, *Ans.*

22. 12 cents. 23. 4 feet.

24. $\frac{1}{2}\sqrt{5}$ and $\frac{1}{4}(\sqrt{5} \pm 5)$.

25. 16. 26. 6 feet.

27. Let $x =$ cost of first horse.

$80 - x =$ cost of second horse.

Then $80 - x =$ gain on first horse.

and $80 - (80 - x) =$ gain on second horse.

$$\frac{80 - x}{x} = \text{rate of gain on first horse.}$$

$$\frac{x}{80 - x} = \text{rate of gain on second horse.}$$

$$\therefore \frac{x}{80 - x} - \frac{80 - x}{x} = \frac{1}{5}.$$

Solving $x = 41.995,$

and $80 - x = 38.005.$

28. The lot is 100 feet × 100 feet = 10,000 square feet. The house and the driveway each covers 5000 square feet.

Let x = the width of the driveway. On each side of the lot it extends from front to rear 100 feet; total, $200\,x$ square feet. The house is 100 feet $- 2\,x$ feet; the driveway behind the house is 100 feet $- 2\,x$ feet long by x feet wide; the total number of square feet is $100\,x - 2\,x^2$. The total number of square feet in the driveway (at sides of lot and rear of house) is $200\,x + 100\,x - 2\,x^2$.

$$\therefore\ -2\,x^2 + 300\,x = 5000.$$

Solving,

$$x = 19.1.$$

29. 672.

30. 7.416 inches from either end.

31. Their monthly wages may be any number of dollars. If they receive more than \$50 a month they will each lay up the same sum. If they receive less than \$50, they will become equally indebted.

32. $\frac{1}{6}$.

33.

$$2(1 + x^4) = (1 + x)^4.$$

$$2 + 2\,x^4 = x^4 + 4\,x^3 + 6\,x^2 + 4\,x + 1.$$

Transposing,

$$x^4 - 4\,x^3 - 6\,x^2 - 4\,x + 1 = 0.$$

Adding $12\,x^2$ to both sides,

$$x^4 - 4\,x^3 + 6\,x^2 - 4\,x + 1 = 12\,x^2.$$

But

$$x^4 - 4\,x^3 + 6\,x^2 - 4\,x + 1 = (x - 1)^4.$$

Then

$$(x - 1)^4 - 12\,x^2 = 0,$$

or

$$(x^2 - 2\,x + 1)^2 - 12\,x^2 = 0.$$

Factoring,

$$(x^2 - 2\,x + 1 + 2\,x\sqrt{3})(x^2 - 2\,x + 1 - 2\,x\sqrt{3}) = 0.$$

$$\therefore\ x^2 - 2\,x + 1 + 2\,x\sqrt{3} = 0,$$

and

$$x^2 - 2\,x + 1 - 2\,x\sqrt{3} = 0.$$

Solving,

$$x = 1 - \sqrt{3} \pm \sqrt{3} - 2\sqrt{3},$$

or

$$1 + \sqrt{3} \pm \sqrt{3} + 2\sqrt{3}.$$

34.
$$x^4 + 4\,m^3x - m^4 = 0.$$

Factoring,

$$(x^2 + mx\sqrt{2} - m^2\sqrt{2} + m^2)(x^2 - mx\sqrt{2} + m^2\sqrt{2} + m^2) = 0.$$

$$\therefore\ x^2 + mx\sqrt{2} - m^2\sqrt{2} + m^2 = 0.$$

Solving, $$x = -\frac{m}{\sqrt{2}} \pm \frac{m\sqrt{2\sqrt{2} - 1}}{\sqrt{2}}.$$

35.
$$x^2 + y = 11. \quad (1)$$
$$y^2 + x = 7. \quad (2)$$

(3) $y - 2 = 9 - x^2$, from (1).

(4) $y^2 - 4 = 3 - x$, from (2).

(5) $y - 2 = \frac{3}{y+2} - \frac{x}{y+2}$, from (4) by dividing by $y + 2$.

$$\therefore\ 9 - x^2 = \frac{3}{y+2} - \frac{x}{y+2}.$$

$9 - \frac{3}{y+2} = x^2 - \frac{x}{y+2}$, by transposing.

Then

$$9 - \frac{3}{y+2} + \left(\frac{1}{2\,y+4}\right)^2 = x^2 - \frac{x}{y+2} + \left(\frac{1}{2\,y+4}\right)^2,$$

completing the square.

$3 - \frac{1}{2\,y+4} = x - \frac{1}{2\,y+4}$, extracting the square root.

Then canceling, $x = 3$,

and substituting, $y = 2$.

NOTE. — From Horner's method we find $x = -2.803,\ 3.581,\ -3.778.$ Hence $y = 3.131,\ -1.849,\ -3.283.$

36. $x = 2;\ y = 1.$

37. $x = 4;\ y = 6.$

38. $x = 2;\ y = 3.$

39. $x = 4;\ y = 9.$

40. $x = 4;\ y = 9.$

41. $x = \pm 2,\ \pm\frac{11}{\sqrt{7}};\ y = \pm 5,\ \pm\frac{19}{\sqrt{7}};\ z = \pm 3,\ \pm\frac{1}{\sqrt{7}}.$

42.
$$5\,y(x^6 + 1) - 3\,x^3(y^2 + 1) = 0. \quad (1)$$
$$15\,y^3(x^2 + 1) - x(y^6 + 1) = 0. \quad (2)$$

$$(3)\quad 15\left(\frac{x^2+1}{x}\right)=\frac{y^6+1}{y^3},\text{ from (2).}$$

$$(4)\quad 5\left(\frac{x^6+1}{x^3}\right)=3\left(\frac{y^2+1}{y}\right),\text{ from (1).}$$

$$(5)\quad 5\left(x^3+\frac{1}{x^3}\right)=3\left(y+\frac{1}{y}\right),\text{ from (4).}$$

$$(6)\quad 15\left(x+\frac{1}{x}\right)=y^3+\frac{1}{y^3},\text{ from (3).}$$

$$(7)\quad x^3+\frac{1}{x^3}=\frac{3}{5}\left(y+\frac{1}{y}\right),\text{ from (5).}$$

$$(8)\quad 3\left(x+\frac{1}{x}\right)=\frac{1}{5}\left(y^3+\frac{1}{y^3}\right),\text{ from (6).}$$

$$\therefore\ x^3+3\left(x+\frac{1}{x}\right)+\frac{1}{x^3}=\frac{1}{5}\left(y^3+\frac{1}{y^3}\right)+\frac{3}{5}\left(y+\frac{1}{y}\right),$$

adding (5) and (6).

Then $$\left(x+\frac{1}{x}\right)^3=\frac{1}{5}\left[y^3+3\left(y+\frac{1}{y}\right)+\frac{1}{y^3}\right].$$

$$5\left(x+\frac{1}{x}\right)^3=\left(y+\frac{1}{y}\right)^3,$$

$$\sqrt[3]{5}\left(x+\frac{1}{x}\right)=y+\frac{1}{y},\text{ extracting the cube root.}$$

But since $$y+\frac{1}{y}=x^3+\frac{1}{x^3}$$

$$\therefore\ x^3+\frac{1}{x^3}=\sqrt[3]{5}\left(x+\frac{1}{x}\right)$$

Dividing by $x+\frac{1}{x}$, we have

$$x^2-1+\frac{1}{x^2}=\sqrt[3]{5}.$$

$$x^2+2+\frac{1}{x^2}=\sqrt[3]{5}+3,\text{ by adding }+3.$$

$$x + \frac{1}{x} = \sqrt{\sqrt[3]{5} + 3}, \text{ by extracting the square root.}$$

Also $$x - \frac{1}{x} = \sqrt{\sqrt[3]{5} - 1}, \text{ by subtracting 1.}$$

$$\therefore\ x = \tfrac{1}{2}[\sqrt{\sqrt[3]{5} + 3} + \sqrt{\sqrt[3]{5} - 1}].$$

But $$y + \frac{1}{y} = \left(x + \frac{1}{x}\right)\sqrt[3]{5} = \sqrt[3]{5} \cdot \sqrt{\sqrt[3]{5} + 3}.$$

$$y^2 - y(\sqrt[3]{5} \cdot \sqrt{\sqrt[3]{5} + 3}) + 1 = 0.$$

$$y = \tfrac{1}{2}[\sqrt[3]{5} \cdot \sqrt{\sqrt[3]{5} + 3} \pm \sqrt{\sqrt[3]{25}(\sqrt[3]{5} + 3) - 4}].$$

$$\therefore\ y = \tfrac{1}{2}[\sqrt[3]{5} \cdot \sqrt{\sqrt[3]{5} + 3} \pm \sqrt{1 + 3\sqrt[3]{25}}].$$

43. Let $x =$ number of feet in one side of the field.

Then, $x^2 - (x - 66)^2 = (x - 66)^2$.

Solving $x = 225.3356^+$ feet.

$\therefore$ he had 50776^+ square feet, *Ans.*

44. 2.93^+ gallons.

45. Let x be the least integral number that will satisfy the conditions; then we shall have

$$x = 39\,y + 25 = 25\,z + 19 = 19\,n + 11;$$

whence, $$z = y + \tfrac{7}{25}\,y + \tfrac{6}{25}.$$

For integral values y must be 17, 42, 67, 92, 117, 142, 167, etc. Hence the value of y that will satisfy the last value of x in third line, is 167; then

$$x = 39\,y + 25 = 25\,z + 19 = 19\,n + 11 = 5369.$$

— From "American Mathematical Monthly."

46. Dr. A saves $\frac{4}{7}$; Dr. B $\frac{9}{13}$; and Dr. C $\frac{14}{19}$. Hence, the chance for one who is dosed by all three is

$$\tfrac{4}{7} \times \tfrac{9}{13} \times \tfrac{14}{19} = \tfrac{72}{247}.$$

47. Let $x =$ Richard's age and y Robin's age.

Then $$2\,x - y + x = 99,$$

and $$2(2\,y - x) = 2\,x - y.$$

$$\therefore\ x = 45 \text{ and } y = 36.$$

48. If the assertion is true, A and B tell the truth and C is mistaken. The chance of this is

$$\tfrac{2}{3} \times \tfrac{6}{7} \times \tfrac{1}{5} = \tfrac{12}{105}.$$

If the assertion is not true, A and B are mistaken and C tells the truth. The chance of this is

$$\tfrac{1}{3} \times \tfrac{1}{7} \times \tfrac{4}{5} = \tfrac{4}{105}.$$

Now, the assertion is true or it is not true, and 12 chances are in favor of its being true to 4 in favor of its being not true. Hence, the probability of its truth is $\tfrac{12}{16}$ or $\tfrac{3}{4}$, *Ans.*

— From "The School Visitor."

49. Let $x =$ weight of the plank, acting through its midpoint with lever arm 7, while the weight of 196 has the lever arm 1; the equation of moments is:

$$7x = 196. \quad \therefore x = 28.$$

50. Let $x =$ number at 5 cents, $y =$ number at 1 cent, $z =$ number at $\tfrac{1}{2}$ cent. Then

$$x + y + z = 100 = 5x + y + \tfrac{1}{2}z.$$

Eliminating z we get the indeterminate equation, $9x + y = 100$.

$$\therefore y = 100 - 9x.$$

This equation gives us eleven integral solutions, as follows:

$x = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.$
$y = 91, 82, 73, 64, 55, 46, 37, 28, 19, 10, 1.$
$z = 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88.$

51. $9\tfrac{3}{5}$.

52. 6.

53. 17 years, 2 months, 2 days.

54. $a + b$.

55. $1 + \sqrt{2} + \sqrt{3}$.

56. $1 + \sqrt{\tfrac{5}{2}} - \sqrt{\tfrac{3}{2}}$.

57. $x^{-\frac{1}{2}} + x^{-\frac{1}{4}} = 6$.

Solving for $x^{-\frac{1}{4}}$, $x^{-\frac{1}{4}} = -\tfrac{1}{2} \pm \sqrt{6 + \tfrac{1}{4}} = 2, \text{ or } -3.$

$$\therefore x^{\ 1} = 16, \text{ or } 81.$$

$$\therefore x = \tfrac{1}{16}, \text{ or } \tfrac{1}{81}.$$

58. $64;\ (-33)^{\frac{6}{5}}$.

59. $a,\ -b$.

60. $x = \frac{8}{3}, \frac{3}{2}$.
$y = \frac{3}{2}, \frac{8}{3}$.
$z = \pm 2$.

61. $x = \frac{a}{2}(1 \pm \sqrt{3})$,
$\frac{a}{2}\left(1 \pm \frac{1}{\sqrt{3}}\right)$,
$y = \frac{a}{2}(1 \mp \sqrt{3})$,
$\frac{a}{2}\left(1 \mp \frac{1}{\sqrt{3}}\right)$.

62. $x = 6,\ -4\frac{1}{2}$.
$y = 12,\ -9$.

63. $x = \sqrt[4]{\frac{1}{2}(\sqrt{2} - 1)}$,
$y = \frac{1}{\sqrt[4]{2(\sqrt{2} - 1)}}$.

64. $x = \frac{5}{2}\sqrt{10 \pm \frac{22}{5}\sqrt{5}}$,
$y = \frac{5}{2}\sqrt{2 \pm \frac{2}{5}\sqrt{5}}$.

65. $x = \frac{1}{2}[\sqrt{\sqrt[3]{3} + 3} + \sqrt{\sqrt[3]{3} - 1}]$,
$y = \frac{1}{2}[\sqrt[3]{3} \cdot \sqrt{\sqrt[3]{3} + 3} \pm \sqrt{3\sqrt[3]{9} - 1}]$.

66. 600 yards.

67. $\frac{n}{3}(n - 1)(2n - 1)$ yards.

68. 6 minutes.

69. Silenus in 3 hours; Dionysius in 6 hours.

70. Let x = time required to overtake B. He travels $20 + 2x$ miles. Hence $\frac{20 + 2x}{x}$ = his rate. Let y = time to go from B to A. He travels $2\sqrt{(100 + y^2)}$ miles. $\frac{2\sqrt{(100 + y^2)}}{y}$ = his rate. After reaching B a second time he has left $10 - x - 2y$ hours to go $2x + 4y$ miles.

$\therefore \frac{2x + 4y}{10 - x - 2y}$ = his rate. But his rate is uniform. Hence we get

$$\frac{20 + 2x}{x} = \frac{2\sqrt{(100 + y^2)}}{y}, \text{ or } 5x^2 = 5y^2 + xy. \qquad (1)$$

$$\frac{20+2x}{x}=\frac{2x+4y}{10-x-2y}, \text{ or } x^2+2xy=50-10y \qquad (2)$$

If $x=vy$ in (1), we get $5v^2-v=5$ or $v=1.10499$.
$\therefore x=1.10499y$. Substituting in (2) gives

$$14.41996y^2+10y=50, \text{ or } y=1.54737.$$

$\therefore x=1.70983$. Rate $=\frac{20}{x}+2=13.69707$ miles an hour.

From "The American Mathematical Monthly."

First Solution:

71. Let $x=$ the part of a man's work the boy does.

$k=$ the number of bushels of apples the man shakes off in a day.

$kx=$ the number of bushels of apples the boy shakes off in a day.

$\frac{kx}{3}=$ the number of bushels of apples each man picks up in a day.

$\frac{kx^2}{3}=$ the number of bushels of apples the boy picks up in a day.

$$\therefore \frac{2kx}{3}+\frac{kx^2}{3}=\frac{4k}{5}.$$

$$\therefore x=0.843909, \text{ about } \tfrac{27}{32} \text{ or } \tfrac{173}{205}.$$

Boy's share of pay = \$10.976; each man's share = \$13.01.

Second Solution:

Suppose a man does x times as much work as a boy.
By the first condition,
Shaking the apples: picking them up $=1:3x$.
By the second condition,
Shaking the apples: picking them up $=\frac{4x}{5}:2x+1$.

$$\therefore 1:3x=\frac{4x}{5}:2x+1, \text{ or } 12x^2-10x-5=0.$$

$\therefore x = 1.185$.

$\therefore$ The money is divided into parts proportional to 1, 1.185, 1.185, and 1.185.

$\therefore$ The boy receives \$10.98 and each man receives \$13.01.

— From "School Science and Mathematics."

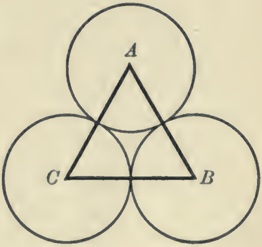
GEOMETRICAL PROBLEMS

1. Construct the $\triangle ABC$, whose base $AB =$ sum of parallel sides, $\angle C =$ angle between diagonals and where $AC + CB =$ sum of diagonals.

Take the point E on AB, such that $CE = CB$; from E draw EF parallel and equal to AC meeting CB in O; join B and F. $CFBE$ is the required trapezoid.

Proof: $AE = CF$, hence $EB + CF =$ given sum of parallel sides. Since $EF = AC$, $EF + CB =$ given sum of parallel sides. And, finally, $\angle EOB = \angle ACB$.

2.



Let $r =$ the radius of the three equal circles.

Then $2r =$ the diameter.

(1) The area of a semicircle whose radius is $r = \frac{\pi r^2}{2}$.

(2) The area of the $\triangle$ is $\frac{\pi r^2}{2} + 200$.

(3) But the area of the equilateral $\triangle$ is $r^2\sqrt{3}$.

$\therefore R^2\sqrt{3} = \frac{\pi r^2}{2} + 200$.

Solving, $r = 498.06$ feet. Then $2r = 996.12$ feet.

3. $6\frac{2}{3}$ feet.

4. 60 feet.

5. Let $R =$ the radius of the sphere, and $2h$ the altitude of the cylinder. Then $R - h =$ the altitude of the segment of the sphere, and $\sqrt{(R^2-h^2)}$ is the radius of the base of the segment and the radius of the cylinder.

The volume of the 2 segments

$$=2[\tfrac{1}{6}\pi(R-h)^3+\tfrac{1}{2}\pi(R-h)(R^2-h^2)],$$

and the volume of cylinder $=2\pi h(R^2-h^2)$.

$\therefore \frac{4}{3}\pi(R^3-h^3)=$ the volume of the segments, and the cylinder $=\frac{2}{3}(\frac{4}{3}\pi R^3)$, by the conditions of the problem.

$\therefore 3h^3=R^3$. $\therefore \frac{4}{3}\pi(R^3-h^3)=\frac{8}{3}\pi h^3=600$, by the conditions of the problem.

$\therefore 2h=2\sqrt[3]{(225/\pi)}$.

6. 1012^+ square feet.

7. 99.379 feet; 11.119 feet.

8. 108.046 feet.

9. 15.708 feet.

10. 13.4316 feet.

11. An isoceles right triangle.

12. 6.18 rods.

13. 3904.

14. 3 feet.

15. Upon the given base *AB* construct a circle whose segment *ACB* shall contain the given vertical angle. Through *E*, the mid-point of *AB*, draw *EF* perpendicular to *AB*, meeting the circumference at *F*. Join *FB*, and perpendicular to *FB* draw *BG* equal to $\frac{1}{2}$ the given bisector of the vertical angle. With *G* as center and *BG* as radius describe the circle *BHL*, and draw *FGL*. With *F* as center, *FL* as radius, describe a circle cutting the given circle in *C*. Join *FC*, cutting *AB* in *D*. Then *ABC* is the triangle required.

In the triangles *FCB* and *FBD*, $\angle FCB=\angle FBA$, since arc $AF=$ arc FB; also $\angle CFB$ is common, hence the triangles are similar, and $FC:FB=FB:FD$; but $FL(=FC):FB=FB:FH$. Therefore $FH=FD$ and $HL=CD$.

Hence in the triangle *ABC*, *AB* is the given base, $\angle ACB$ the given vertical angle, and *CD* the given bisector, and the triangle is satisfied in every condition.

— From "American Mathematical Monthly."

16. $\frac{1}{3}$.

17. Height = radius.

18. $\frac{5}{11}$.

19. 12.91 miles.

20. Let $x =$ a side of the equilateral triangle. Also a, b, and $c =$ the given distances from the point to the sides.

(1) The area of the equilateral $\triangle = \frac{x^2}{2}\sqrt{3}$.

(2) The area of the equilateral $\triangle = \frac{x}{2}(a + b + c)$.

$\therefore \frac{x^2}{2}\sqrt{3} = \frac{x}{2}(a + b + c)$. Solving, $x = \frac{a + b + c}{\sqrt{3}}$.

21. 10341.1 cubic inches.

22. 16.9704.

23. Three inches solid is greater, for
Three solid inches = 3 cubic inches.
Three inches solid = 27 cubic inches.

24. Their homes will be the vertices of an equilateral triangle, and hence the well must be dug where the bisectors of the angles meet.

26. 600 square feet.

27. 8.0558.

29. 7 feet.

30. 10 feet.

33. 39.79 cubic inches.

34. Diameter of the fixed circle.

35. 4 feet.

36. 10 feet.

37. 4330.13 square inches.

38. 10,000 square inches.

39. 17204.77 square inches.

40. 259.81 square feet.

41. 363.39 square feet.

42. 482.84 square feet.

43. 618.182 square feet.

44. 769.421 square feet.

45. 936.564 square feet.

46. 1119.615 square feet.

47. $2\frac{6}{7}$ feet.

48. $16\frac{2}{3}$ inches.

49. 1.755 inches.

50. 10.198 feet.

51. 10.863 inches.

52. 10.142 ft.

53. $33\frac{1}{3}$ feet.

54. 71.344 feet from smaller; 70,071 feet from larger.

55. 15.38756 feet.

56. 2 feet.

57. 1.84 cubic feet.

58. $126\frac{9}{16}$ square inches.

59. 122.84 square inches.

60. 11.66 square inches.

61. 251.328 square feet.

62. 6.8068 cubic feet.

63. 4.192 feet from large end.

65. 154.9856.

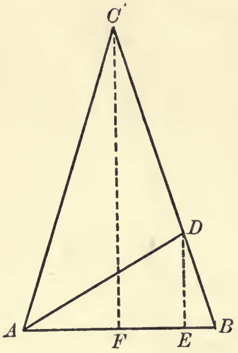
67. 2106 square yards.

68. $80=$ base; $60=$ altitude.

69. 101 feet.

70. 29.41 rods.

73. 78.572 feet.



74. Let $a = BC$ and AC, the ladders; $BD=10$; and $BF=7$; then $AD=a-10$. Let $x=BE$; then $AE=14-x$; and we have $\sqrt{(ED^2+EA^2)}=a-10$. But $ED^2=10^2-x^2$, and $EA^2=(14-x)^2$. Hence $\sqrt{[10^2-x^2+(14-x)^2]}=a-10$.

Also, $7 : x = a : 10$, whence $x=70 \div a$. Substituting and reducing,

$$a^3-20a^2-196a+1960=0;$$

$$\therefore\ a=24.72189 \text{ feet, } \textit{Ans.}$$

75. 78 feet.

80. 13.65 rods.

81. Let PA' and $B'C'$ intersect at K. Draw through K a line parallel to BC cutting AB at x and AC at y. $\triangle PB'C'$ is isosceles; therefore $\angle B' = \angle C'$. The points P, K, y, B' are concyclic; hence $\angle y = \angle B'$. The points P, x, C', K are concyclic; hence $\angle x = \angle C'$. $\therefore \angle x = \angle y$, and $\triangle Pxy$ is isosceles. Hence K is the mid-point of xy, since PK is perpendicular to xy. $\therefore$ K is on the median through A.

— From "School Science and Mathematics."

82. In $\triangle ABC$ let m be the bisector of the $\angle A$, and n the bisector of $\angle B$.

Then,

$$m = \frac{2}{b+c}\sqrt{b \cdot c \cdot s(s-a)}, \text{ and } n = \frac{2}{a+c}\sqrt{a \cdot c \cdot s(s-b)}.$$

See Schultze and Sevenoak's "Geometry," page 158.

$$\therefore \; \frac{2}{b+c}\sqrt{b \cdot c \cdot s(s-a)} = \frac{2}{a+c}\sqrt{a \cdot c \cdot s(s-b)},$$

or

$$\frac{b(s-a)}{(b+c)^2} = \frac{a(s-b)}{(a+c)^2} \qquad (1)$$

Replace s by $\frac{1}{2}(a+b+c)$, simplify, and factor, equation (1) becomes

$$(b-a)[c^3 + c^2(b+a) + 3\,abc + ab(b+a)] = 0.$$

Since the second factor cannot be zero, $b - a = 0$, and $a = b$.

106. 5 inches.

MISCELLANEOUS PROBLEMS

1. 10,945.

2. 221.995 cubic inches. Solve by using the formula,

$$V = r^2\sqrt{3}(\pi e - 2\,r\sqrt{2}), \text{ where } e \text{ is the edge.}$$

3. 367 trees. **4.** 84.823 square feet; 63.617 cubic feet.

5. $21\frac{1}{3}$ feet. **6.** $88\frac{8}{9}$ square feet. **7.** $8\frac{2}{3}$ square feet.

8. 436.21 cubic inches. **9.** 362.8167 square feet.

10. Let $r =$ the radius of the ball. Then $(r-4)$ will be the radius of the hollow sphere inclosed by the shell.

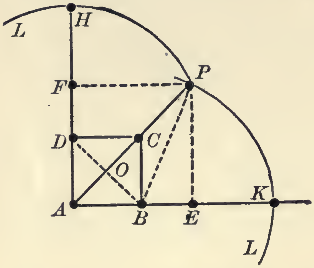
As the volumes of spheres are proportional to the cubes of their radii, the conditions of the problem require that

$$r^3 - (r-4)^3 = \tfrac{1}{5}\,r^3, \text{ or } \tfrac{4}{5}\,r^3 = (r-4)^3.$$

$$\therefore r = \frac{4}{1 - \sqrt[3]{\frac{4}{5}}} = 55.79^{+} \text{ inches, } \textit{Ans.}$$

11. 15.29 feet. **12.** 64 feet. **13.** 18.62 feet.

14. The sparrow flies $66\frac{2}{3}$ feet, the eagle $133\frac{1}{3}$ feet.



15. Let $AB = 25$; $AC = 25\sqrt{2}$; $OB = \frac{25}{2}\sqrt{2}$.

$$BK = 100 - 25 = 75;$$

$$PO = \sqrt{BP^2 - OB^2} = \tfrac{25}{2}\sqrt{34}.$$

$$AP = \tfrac{25}{2}(\sqrt{34} + \sqrt{2});$$

$$AE = \frac{AP}{\sqrt{2}} = \frac{25}{2}(\sqrt{17} + 1).$$

Area $AEPF = AE^2 = \frac{625}{2}(9 + \sqrt{17})$.

Area $ABCD = 25^2 = 625$,

and area $BEPFDC = \frac{625}{2}(9 + \sqrt{17} - 2) = 3476$ square feet.

$$EK = 100 - AE = \tfrac{25}{2}(7 - \sqrt{17}).$$

Area of segments PFH and PEK

$$= \frac{\overline{EK}^3}{4\,PE} + \tfrac{2}{3}(2\,PE \times EK) = 3252 \text{ square feet.}$$

Area $AHLK = \frac{3}{4}\pi \times 100^2 = 23{,}562$ square feet.

$23{,}562 + 3252 + 3476 = 30{,}290$ square feet.

— From "The School Visitor."

16. In order to overturn the cube it must be revolved on a lower edge until the center of mass is vertically over that edge, and this will require the lifting of the 300 pounds through a distance $a(\sqrt{2} - 1)$, a being the edge of the cube against gravity.

$\therefore$ the work done $= 300\,a(\sqrt{2} - 1) = 124.26\,a$ foot pounds. Hence, the size of the cube cannot be left out of the calculation.

— From "The American Mathematical Monthly."

17. 34.6785 feet. **18.** 4.72 rods. **19.** 7.92 rods.

20. 22.72 feet. **21.** 76.394 feet.

22. 11.9206; 8.0794; 8.0794; 11.9206.

23. 18.2948 feet.

24. 38.5704.

25. 124.905 feet.

26. 11.817 inches.

27. 1.87$^+$ feet.

28. 889.337$^+$.

29. 24,630.144 acres.

30. 2765.45 square yards.

31. Two feet from the end of the log.

32. 249.03 inches.

33. 16.125 square inches.

34. 108 sheep.

35. The rabbit goes 133⅓ yards; the hound goes 166⅔ yards

36. 128.

37. 180.

38. 19.8; 35.7; 44.5.

39. 16.2484 cubic inches.

40. 60°.

41. 113.0976 square feet.

42. 7.2 inches.

43. 11.34 feet per second.

44. 90 pounds.

45. 350.163 square yards.

46. 2467.4 cubic feet.

47. 602.349 cubic inches.

48. 355.88 square inches.

49. 6830.47 cubic inches.

50. 842.044 square inches; 404.318 cubic inches.

52. 125.6638 square feet; 31.4159 cubic feet.

53. 1,184,352.528 cubic inches.

54. Any force greater than 202½ pounds will draw the wheel over the log.

55. 19.7392 cubic feet.

56. 39.4784 square feet.

57. 4421.58 square inches; 17,686.32 cubic inches.

58. 1473.86 square inches; 3457.92 cubic inches.

59. 372.30 cubic feet.

60. 27.12 cubic inches.

61. .596$^+$ feet.

62. 36 square feet.

63. $42\frac{2}{3}$ cubic feet.

64. 226.2 cubic inches.

65. $7.61\frac{2}{3}$ square feet.

66. 189.8 inches.

67. 628.32 square inches.

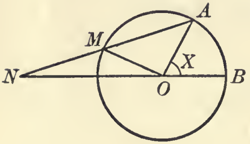
68. 400 square inches.

69. 339.29 square feet.

70. A, 44.69828^{+} mi.; B, 86.81897^{+} mi.

71. 79.119 feet.

72. Let X, or AOB be the given angle.



With O as a center and any radius, describe a circle.

Draw the secant AMN, making MN equal to the radius of the circle.

(This can be done only by using a graduated ruler.)

Join the points O and M.
Then $\angle N = \frac{1}{3}\angle X$.

Proof.

$$\angle MNO = \angle MON.$$

$$\angle AMO = \angle N + \angle MON = 2 \times \angle N.$$

$$\angle AMO = \angle MAO.$$

$$\angle N + \angle MAO = \angle X.$$

$$\therefore \angle N = \tfrac{1}{3}\angle X.$$

For other solutions, see Ball's "Mathematical Recreations."

73. For 20 pounds on 10 arm, weight $= \dfrac{20 \times 10}{9} = 22\frac{2}{9}$.

For 20 pounds on 9 arm, weight $= \dfrac{20 \times 9}{10} = 18$.

$22\frac{2}{9} + 18 = 40\frac{2}{9}$ pounds or $\frac{2}{9}$ pounds he loses.

$\frac{2}{9} \div 40 = \frac{1}{180}$, $\frac{1}{180}$ of 100% $= \frac{5}{9}$% he loses.

—From "School Science and Mathematics."

74. Let $x =$ pressure on B's shoulder.

The moments about A's shoulder are 5×54 down, and $9x$ up.

$\therefore 9x = 5 \times 54$. $\therefore x = 30$, weight on B's shoulder.
In like manner we may let $x =$ the pressure on A's shoulder.
Then $9x = 4 \times 54$. $\therefore x = 24$, weight on A's shoulder.

75. Since the momenta of the bullet and gun are equal in magnitude, $7v = \frac{1}{32} \cdot 1400$, whence $v = 6.26$ foot seconds and $E = \frac{mv^2}{64.2} = \frac{7 \cdot 6.26^2}{64.2} = 4.3 =$ energy of recoil.

Also $W = Fs$. $\therefore 4.3 = \frac{4}{12} F$, or $F = 12.9$ pounds.

76. Let w equal energy of ball, and v its velocity on emergence. Then $w = 1000^2 \times \frac{m}{n}$. Energy after passing through plank is $\frac{5}{6} w = \frac{vw^2}{2}$.

$\therefore v^2 = \frac{5}{6} \times 1000^2$, or $v = 912.87$ feet per second.

77. 16,956.1 square feet.
78. 213,825.15 acres.
79. 989.96 feet.
80. 68,948.77 feet.
81. 337.5 cubic feet.
82. 22.386 feet.
83. 41¼ feet.
84. 40 rd. = length; 30 rd. = breadth; 7½ acres = area.
85. 80 rd. = length; 60 rd. = breadth.
86. 100 feet.
87. 50 rods.
88. 48 inches.
89. 36.57 acres.
90. 31,416 square feet.
91. 6 feet; 8 feet; 10 feet.
92. $.80449^+$.

MATHEMATICAL RECREATIONS

1. Let $x =$ difference between Mary and Ann's ages.
Then $24 - x =$ Ann's age.
Therefore $12 + x = 24 - x$.
$\therefore x = 6$.
$\therefore 24 - x = 18$, Ann's age.

2. 20 pounds.

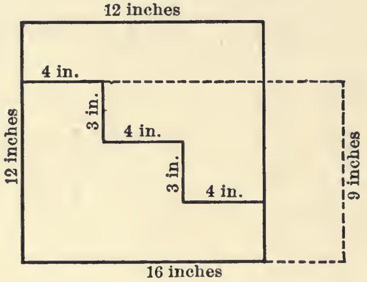
3. Take the six matches and form a tetrahedron. This tetrahedron will have four faces, each face being bounded by three matches which form an equilateral triangle.

4. The same.

5. To find the digit crossed out subtract the remainder from the next highest multiple of nine.

6. To find the figure struck out subtract the remainder from the next highest multiple of nine.

7.

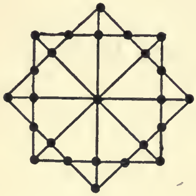


8. $99\frac{9}{9}$.

9. In forming the second figure from the four parts of the first the lines forming squares do not coincide exactly, thus seemingly forming 65.

10. The blacksmith was right, as he had to cut and weld only three links.

13.



14. 25; 15; 20.

15. 40 feet

16. Rides 3; walks 1.

18. ∞ %

19. $34\frac{1}{2}$; $31\frac{1}{2}$.

20. There will be no lot, since the given dimensions will not make a triangle.

21. None.

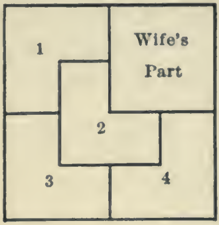
22. 5 and 6 are 11.

23. A, $25.53; B, $19.15; C, $15.32.

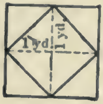
24. They borrowed one sheep, which made 18. After dividing they had one left, which was returned to the owner.

25. Only 6 cats.

26.



27.



28. The pickets, standing vertically, are supposed to be uniformly the same distance apart at the base; practically there would be the same number as at the top of the elevation, if these pickets were extended downward to a common level.

29. 8 cats.

30. In each case the middle digit is 9 and the digit before it (if any) is equal to the difference between 9 and the last digit.

31. Subtract 14 from the result given, and obtain a number of two digits which are the numbers originally chosen. The digit in tens' place is the number that was multiplied by 5.

33. If the second remainder is less than the first, the figure erased is the difference between the remainders; but if the second remainder is greater than the first, the figure erased equals 9, minus the difference of the remainders.

34. I make my additions so that the sums are respectively, 12, 23, 34, 45, 56, 67, 78, and 89.

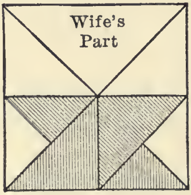
35. $8 and the boots.

36. Take the goose over, return and take the corn over, bring the goose back, take the fox over, then return for the goose.

37. Gravity would cause the ball to descend toward the center of the earth with an increased velocity, but coming constantly to a point of less motion in the earth, it would soon scrape on the east side of the hole, until it passed the center, where it would be constantly passing points having a faster motion than the center; it would soon scrape on the opposite side; the friction thus retarding the motion, it would pass and repass the center of the earth until it would finally come to rest at this point.

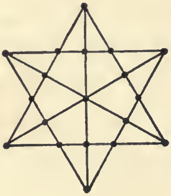
— From "Curious Cobwebs."

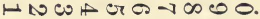
38.



39. With a one, a three, a nine, and a twenty-seven pound weight.

40.



41. Sometimes, .

42. If so, by the same logic, you can multiply eggs by eggs and get square eggs, or multiply circles by circles and get square circles. It is impossible to multiply feet by feet for the principles of multiplication are — (1) The multiplier must be regarded as an abstract number. (2) The multiplicand and product must be like numbers.

43. No.

45. I will always have money.

46. (I) *R* pushes *P* into *A*. (II) *R* returns, pushes *Q* up to *P* in *A*, couples *Q* to *P*, draws them both out to *F*, and then pushes them to *E*. (III) *P* is now uncoupled, the engine takes *Q* back to *A*, and leaves it there. (IV) The engine returns to *P*, pulls *B* back to *C*, and leaves it there. (V) The engine running successively through *F*, *D*, and *B*, comes to *A*, draws *Q* out, and leaves it at *B*.

— From Ball's "Mathematical Recreations."

47. Place 5 on 4, 2 on 1, 11 on 10, and 8 on 7.

48. Fill the 3-gallon cask and pour it into the 5. Fill it again and pour into the 5 until the 5 is full. There is now 1 gallon left in the 3. Pour back the 5 into the 8, and the one gallon left in the 3 into the 5. Then fill the 3 and pour into the 5, making 4 gallons in the 5-gallon cask, or one half of the 8 gallons.

49. $20.

53. $80.6\dot{9} + .7\dot{4} + .\dot{5}$.

54. $\frac{148}{296} + \frac{35}{70}$.

55. $78 + 15 + \sqrt[2]{9} + \sqrt[3]{64}$.

56. $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 0 = 0$.

57. $\dfrac{35}{70} + \dfrac{\sqrt[2]{4 \times 9}}{6} + 1^8$.

58. Suppose he starts from *F*. Then he may take either of two routes.

(1) *F B A U T P O N C D E J K L M Q R S H G F.*

(2) *F B A U T S R K L M Q P O N C D E J H G F.*

Rule. *The route from any town may be found by either of the following rules, in which r denotes he is to take the road to the right, and l denotes that he is to take the road to the left.*

(1) *r r r l l l r l r l r r r l l l r l r l.*

(2) *l l l r r r l r l r l l l r r r l r l r.*

59. Second class. The hand is the *P*, the boat the *W*, and the water the *F*.

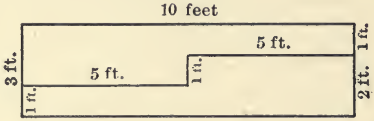
60. \$.75 \$75. **63.** 28 eggs. **64.** 32^{+} feet. **65.** \$2.50.

66. They sell 49, 28, and 7 at the rate of 7 for a cent; then 1, 2, and 3 at 3 cents each; hence each one receives 10 cents.

67. 39.79^{+} pounds. **69.** See No. 48.

70. Answer, 21. It cannot be greater than the smallest number, 27; it cannot be 27, since the remainders would then be different. By dividing these numbers, one by another, 48 by 27, 90 by 48, 174 by 90, we find the remainders to be 21, 42, 84, the last two being multiples of the first. Now dividing the numbers by these remainders (21 and the multiples), 27 by 21, 48 by 42, 90 by 84, and 174 by 168, the next multiple of 21, we obtain a remainder which is the same in each case; we therefore conclude that dividing all the numbers by 21 would give a like result.

71



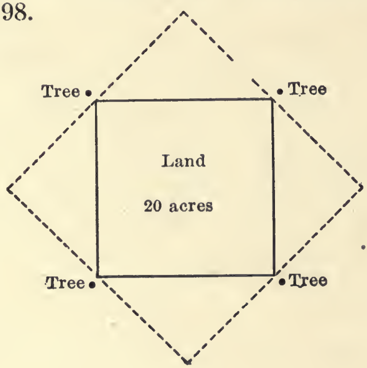
72. $\frac{1}{2}$. **73.** XIX. Take the 1 away and have XX.

74. $1\frac{1}{2}$ pounds.

75. Invert the 6 to make it 9. The whole number may be either 918 or 198.

76. 1.25.

77.



78. The method would have been incorrect. The division would have been in favor of the tenant. The landlord would have received $\frac{2}{5}$ of 45 bushels when he was entitled to $\frac{2}{5}$ of the 45 bushels and also to $\frac{2}{5}$ of the 18 bushels. In other words, he should have received $\frac{2}{5}$ of $(45 + 18)$, or $25\frac{1}{5}$ bushels. The landlord would have lost the difference between $25\frac{1}{5}$ bushels and 18 bushels, or $7\frac{1}{5}$ bushels.

79. 792. **80.** 20. **81.** 0.

82. By immersing them in a vessel of water.

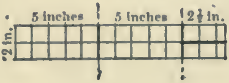
83. B hoes six the most. **84.** 3. **85.** $3^3 + 3$.

86. IX = 9. Cross the I and make it XX.

87. 21 days, because two of the ears are his own.

88. 43 days. **89.** 64

90.



91. First, the two sons cross, then one returns. Second, the man crosses and the other son returns. Third, both sons cross, and then one returns. Fourth, the lady crosses and the other son returns. Fifth, the two sons cross.

92. 199. **93.** Infinity. **94.** $2\frac{2}{3}$.

95. $7\frac{1}{2}$. **96.** $3^3 - 3$, or $22 + 2$. **97.** 29 days.

98. *Ans.* 987,654,321.

When multiplied by 18 = 17,777,777,778.
When multiplied by 27 = 26,666,666,667.
When multiplied by 36 = 35,555,555,556.
When multiplied by 45 = 44,444,444,445.
When multiplied by 54 = 53,333,333,334.
When multiplied by 63 = 62,222,222,223.
When multiplied by 72 = 71,111,111,112.
When multiplied by 81 = 80,000,000,001.

When multiplied by 99 = 97,777,777,779.
When multiplied by 9 = 8,888,888,889.
When multiplied by 90 = 88,888,888,890.

The same is true of higher multiples of nine. Thus,

$$108 \times 987{,}654{,}321 = 106{,}666{,}666{,}668.$$

$$117 \times 987{,}654{,}321 = 115{,}555{,}555{,}557.$$

99. Three cents for each seven and nine cents each for the remainder.

100. It will make no difference as long as he jumps on the deck. Should he jump off the boat, then the effect would be different.

101. 10. **102.** 3^3.

103. When the figures added make nine or some multiple of nine.

104. $3+0+2+0+1+1=7$. $9-7=2$. Two is wanting to make a multiple of nine, therefore 2 placed anywhere in, or before, or after the number, will make it divisible by nine.

105. 11 grooms and 15 horses. **106.** 11 cents.

107. 301. **108.** 300 pounds; also 300 pounds.

109. Because muscles and bones are heavier than fat. The specific gravity of a fat man is therefore less than that of a lean one.

110. The ball which is thrown has time to impart its motion to the board; but the one fired has not.

111. Move from 1 to 6, 4 to 1, 7 to 4, 2 to 7, 5 to 2, 8 to 5, and 3 to 8.

112. 8888, when halved equals 0000.

113. $4 \pm 2\sqrt{3}$.

114. May have any shape; square.

115. 1 is the only integer. This is however true of any number between 0 and 2.

116. $2+2=2^2$, or $0+0=0^2$.

117. 142,857.

118. 12.

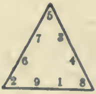
121.
$$\begin{array}{l} 64 \\ \underline{25} \\ 89+1+3+7=100. \end{array} \qquad \begin{array}{r} 15 \\ 36 \\ \underline{47} \\ 98 \\ \underline{2} \\ 100 \end{array} \qquad \begin{array}{r} 56 \\ 8 \\ 4 \\ \underline{3} \\ 71 \\ \underline{29} \\ 100 \end{array} \qquad \begin{array}{r} 95\frac{1}{2} \\ \underline{4\frac{38}{76}} \\ 100 \end{array}$$

Many other solutions.

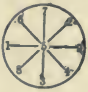
122.

6	10	3	15
11	7	14	2
16	4	9	5
1	13	8	12

123.

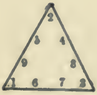


124.



125. Arrange the figures as in (12) except use such figures to place opposite each other that when added make 20. Use 10 at center.

126.



127.

8	1	6
8	5	7
4	9	2

129. 7.

132. B, C, and A respectively.

134. (1) $32043^2 = 1026753849$.

(2) $99066^2 = 9814072356$.

(3) (4) The least solutions which have been found of (3), (4) are identical:

$10101010101010101^2 = 10203 \cdots 080908 \cdots 30201$, but there are probably lower numbers suitable. If numbers beginning with zero were admissible, then much lower numbers would suffice, *e. g.*, $01111111110^2 = 001234567898765432100$.

—From "Mathematical Reprint."

135. 25.3 rods.

136. Let $x =$ A's ability and $y =$ B's ability. Since A can dig the ditch in the same time that B shovels the dirt, $x : y =$ the labor required to dig it : the labor required to shovel the dirt.

And since B can dig twice as fast as A can shovel, $\frac{1}{2}y : x =$ the labor required to dig it : the labor required to shovel the dirt.

$\therefore x : y = \frac{1}{2}y : x$.

$\therefore y = x\sqrt{2} = 1.414x$.

A should receive $\frac{x}{x+y}$ of \$10, or $\frac{x}{x+1.414x}$ of $\$10 = \frac{1}{2.414}$ of $\$10 = \$4.14$. B received \$5.86.

138. This is a mere trick. When trains meet they must be at the same distance from a given point.

139. 0 is the only possible digit to satisfy the first addend total. 2 being "carried" to the second column, 2 is found to be the next missing number. The third column total must be either 23 or 33. On trial, the former is found to be wrong, and the only two numbers making 18, so as to give 33 as total of third line, are 9 and 9. Proceeding, we find that 9 is wanted in the fourth line, and in the sixth line two 0's. Only these numbers will satisfy, and the answer is proved by adding.

140. The only number to satisfy the product by 5 is 3, and this being supplied the remaining numbers are easily found.

The only possible numbers, in the multiplier are 3 and 8; of these we see that 8 is the one required. The third missing number is, of course, 6.

141. If the question be put down in skeleton, we shall be more readily able to supply the missing links.

```
)529565(***
 ****xx
 ------
  2466
   ****  → 2244
   -----
   2225
    ****  → 1683
   -----
    542
```

The last remainder being 542, we see that 2225 – 542, *i.e.* 1683, must be the last multiple of the divisor. Similarly, 2466 – 222 leaves 2244 as another multiple. The G. C. M. of 2244 and 1683 is 561, and this number is greater than the largest remainder (542), hence 561 is the divisor. The quotient, by division, will be found to be 943.

142. The middle digit remains unaltered, and since in adding the second digit is 7 (1 being carried), the second line total must be 17, and therefore 8 must be the middle digit. Again the first digit total must be 10, and the only possible addends are 9 and 1, 8 and 2, 7 and 3, 6 and 4, 5 and 5. By testing it will be seen that 9 and 1 are the only two numbers fulfilling conditions. *Ans.* 981 and 189.

143. Instead of multiplying by 409, she actually multiplied by 49, therefore her answer was short of the true answer by (409 – 49), *i.e.* 360, times the multiplicand, and we are told this was 328,320; 328,320 ÷ 360 = 912, the multiplicand.

— From "Arithmetical Wrinkles."

144. 3 ft.

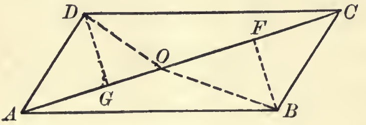
145. (1) Webster's Dictionary says that after the sign IↃ for D, the character Ↄ (called the apostrophus) was repeated one or more times, each repetition having the effect of multiplying IↃ by 10. To represent a number twice as great, C was repeated as many times before the stroke, I, as the Ↄ was given after it. Hence, one billion is written,

CCCCCCCIↃↃↃↃↃↃↃ

(2) A bar placed over any number in the Roman notation multiplies the original value by 1000; hence two bars placed over it would be a thousand times a thousand times its initial value, and $\overline{\overline{\mathrm{M}}} = 1000 \times 1000 \times 1000 = 1{,}000{,}000{,}000$.

—From "The School Visitor."

146. Let AC locate the ditch and O the point required. The triangles AOD and COB have equal altitudes, DG and BF; hence, their bases AO and OC must be to each other as their areas, or as 2 to 3, and O may be $\frac{2}{5}$ of the distance from A to C or from C to A.



147. Let x, y, and z be the three digits. Then, $(100x+10y+z)-(100z+10y+x)=99x-99z$. Consequently, the difference for any set of three digits is $9 \times 11\,(x-z)$.

The result is always 99 times the difference of the extreme digits.

148. $99\frac{99}{99}$.

149. Multiply the selected number by nine, and use the product as the multiplier for the larger number. It will be found that the result is in each case the "lucky" number, nine times repeated.

150. The father was three times the age of his son $15\frac{1}{2}$ years earlier, being then $55\frac{1}{2}$ while his son was $18\frac{1}{2}$. The son will have reached half his father's age in 3 years' time, being then 37, while his father will be 74.

151. 45.

152. The asterisks indicate the figures to be expunged.

$$\begin{array}{r} *\,11 \\ 33\,* \\ *** \\ 77\,* \\ *** \\ \hline 1111 \end{array}$$

153. By the conditions there were twelve children in all and each has now nine, then each parent had three children when married, making six arrivals within ten years.

156.

$$\begin{array}{r} 50\frac{1}{2} \\ 49\frac{38}{76} \\ \hline 100 \end{array} \qquad \begin{array}{r} 80\frac{27}{54} \\ 19\frac{3}{6} \\ \hline 100 \end{array}$$

Many other solutions.

157.

$1 = 44 \div 44.$

$2 = 4 \div 4 + 4 \div 4.$

$3 = (4 + 4 + 4) \div 4.$

$4 = \sqrt{(4 \times 4 \times 4 \div 4)}.$

$5 = \sqrt{(4 \times 4)} + 4 \div 4.$

$6 = 4 \div .4 - \sqrt{(4 \times 4)}.$

$7 = 44 \div 4 - 4.$

$8 = (4 + 4) \times (4 \div 4).$

$9 = 4 + 4 + 4 \div 4.$

$10 = 4 \div .4 + 4 - 4.$

$11 = 4 \div .4 + 4 \div 4.$

$12 = 4 \times 4 - \sqrt{(4 \times 4}.$

$13 = 44 \div 4 + \sqrt{4}.$

$14 = 4 \div .4 + \sqrt{(4 \times 4)}.$

$15 = 44 \div 4 + 4.$

$16 = 4 \times 4 - 4 + 4.$

$17 = 4 \times 4 + 4 \div 4.$

$18 = 4 \div .4 + 4 + 4.$

$19 = (4 + 4 - .4) \div .4.$

$20 = 4 \div .4 + 4 \div .4.$

$21 = (4.4 + 4) \div .4.$

$22 = (4 + 4) \div .4 + \sqrt{4}.$

$23 = 4 \div (.4 \times .4) - \sqrt{4}.$

$24 = (4 + 4) \div .4 + 4.$

$25 = (4 + 4 + \sqrt{4}) \div .4$

$26 = 4 \times 4 + 4 \div .4.$

$27 = 4 \div (.4 \times .4) + \sqrt{4}.$

$28 = 44 - 4 \times 4.$

$29 = 4 \div (.4 \times .4) + 4.$

$30 = (4 + 4 + 4) \div .4.$

158. The figure that occurs in the quotient is the difference between the first and last figures of the number taken.

159. The figure erased is the first remainder minus the second, or if the first is not greater than the second, then it is the first $+9-$ the second.

160. Yes. For example $\frac{-2}{+5}=\frac{+4}{-10}$.

161. Every even number contains 2 as a factor and every alternate even number contains 4 as a factor; hence, the product of any two consecutive even numbers is divisible by 8.

162. Indeterminate.

163. A gets seven ninths as much as B per rod; hence to get equal money A must dig nine sevenths as much. Dividing 100 rods in proportion of 9 to 7, A must dig 56.25 rods and B 43.75 rods. For actual work each gets thus an equal sum, $98.4375, and we may now infer that the balance of the money should be equally divided, giving each $100.

164. 3 ounces. **165.** 10 cents.

166. There is no change in the weight, since the weight of the fish is the same as the weight of the water displaced.

167. A rectangular tank twice as wide as it is deep with a square base.

168. $13.75.

169. 300.

170. The bird is heavier than the air and supports itself by striking down upon the air. The increase in weight caused by these strokes would undoubtedly be the difference between the weight of the bird and the weight of its displacement of air.

171. Suppose John's rate of work is w times James'.

Then, by the first condition,

$$\text{James' work} : \text{John's work} :: 3 : w,$$

and by the second condition,

$$\text{James' work} : \text{John's work} :: w : 1.$$

$\therefore 3 : w :: w : 1.$

Then $w = \sqrt{3}$, a mean proportional.

Then \$10 must be divided between John and James in the ratio of $1 : \sqrt{3}$, which makes John's share \$3.66, and James' share \$6.34.

173. 5.

174. Subtract from the higher multiple of 9.

176. A mile square can be no other shape than square; the expression names a surface of a certain specific size and shape. A square mile may be of any shape; the expression names a unit of area, but does not prescribe any particular shape.

179. 36 cents.

180. 2.

181.

$$\begin{array}{rrr} \text{SIX} & \text{IX} & \text{XL} \\ \text{IX} & \text{X} & \text{L} \\ \hline \text{S} & \text{I} & \text{X} \end{array}$$

182. 41.78$^+$ feet.

183. The correct answer is $\frac{1}{4}$.

185. There are several solutions. One is as follows:

The vessels can hold	24 oz.	13 oz.	11 oz.	5 oz.
Their contents to begin with are	24	0	0	0
First, make their contents	0	8	11	5
Second, make their contents	16	8	0	0
Third, make their contents	16	0	8	0
Fourth, make their contents	3	13	8	0
Fifth, make their contents	3	8	8	5
Last, make their contents	8	8	8	0

186. He lost.

187. $\frac{240}{253}$.

Let $\frac{x}{y}$ be the required fraction.

Then $\frac{x}{y}$ pounds $+ \frac{x}{y}$ shillings $+ \frac{x}{y}$ pence $= 1$ pound.

Reducing all to pence, we have,

$$\frac{240x + 12x + x}{y} = 240.$$

$$\therefore \frac{253x}{y} = 240.$$

$$\text{Solving, } \frac{x}{y} = \frac{240}{253}.$$

188. The number 45 is the sum of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9. The puzzle is solved by arranging these in reverse order, and subtracting the original series from them, when the remainder will be found to consist of the same digits in a different order, and therefore making the same total.

Thus,
$$\begin{array}{r} 987654321 = 45 \\ \underline{123456789 = 45} \\ 864197532 = 45 \end{array}$$

191. One traveled ten times around the world, and the other remained at home, or both traveled around the earth in opposite directions, the sum of the two sets of circumnavigation amounting to ten.

192. (*a*) They start in a high latitude and on the same meridian, both going east or west. (*b*) They start in a high latitude, both on the same parallel and travel south (or if in a high southern latitude they would travel north). (*c*) They may start each ten miles from the north pole 180° of longitude apart, and each travels five miles south.

193. They travel from the North Pole to the South Pole, or from the South Pole to the North Pole.

194. Standing on the North Pole.

195. It will never come up.

196. 40 pounds.

197. $(9\frac{9}{9})^{\frac{9}{9}}$; $9\frac{99}{99}$; ~~$9.99\frac{9}{9}$~~; $\sqrt[9]{9^9} + \frac{9}{9}$.

200. All that is necessary is to deduct 25 from the sum named. This will give a remainder of two figures, representing the points of the two dice.

201. Subtract 250 from the sum named. This will give a remainder of three figures, representing the points of the three dice.

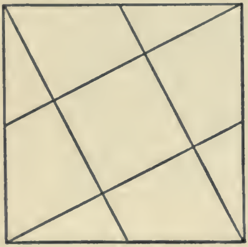
202. I Solution. Sam and John each take 2 full casks, 2 empty casks, and 3 half-full casks; and James, 3 full casks, 3 empty casks, and 1 half-full cask.

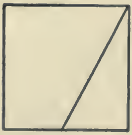
II Solution. Sam and John each take 3 full casks, 1 half-full cask, and three empty casks; and James 1 full cask, 5 half-filled casks, and 1 empty cask.

204. This problem is susceptible of various answers, equally correct, according to the value assigned to the smallest part, or unit of measurement. If this unit of measurement be 1, the number will be $1+40+400+500=941$. If the unit be 2, the number will be $2+80+800+1000=1882$, and so on *ad infinitum.*

205. 2519.

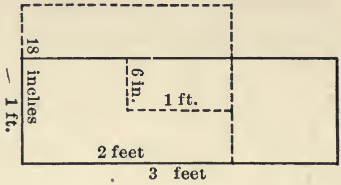
206. Find the center of either side of a given square, and cut the card in a straight line from that point to one of the opposite corners, as shown in the small figure. Treat four of the five squares in this manner. Rearrange the eight segments thus made with the uncut square in the center, as shown in the larger figure, and you will have a single perfect square.

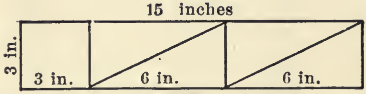


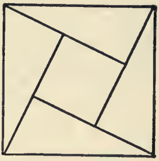


—From "Mechanical Puzzles."

207.

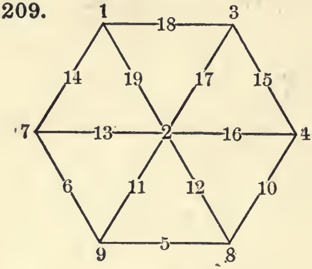

18 inches
1 ft.
6 in.
1 ft.
2 feet
3 feet

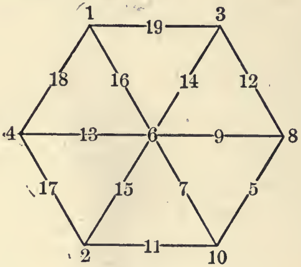

15 inches
3 in.
3 in.
6 in.
6 in.



208. Cut as indicated in first figure, and rearrange the pieces as shown in the second figure.

209.


1
18
3
14
19
17
15
7
13
2
16
4
6
11
12
10
9
5
8

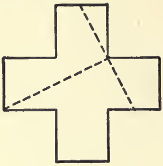

1
19
3
18
16
14
12
4
13
6
9
8
17
15
7
5
2
11
10

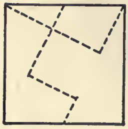
210. Count and mark every ninth one, marking it "Turk" until 15 are marked. Mark the remaining ones, "Christians."

211. $16\frac{5}{8}$. **213.** 7 and 5. **214.** $10\frac{2}{7}$ hours. **217.** 60.

215. 72. **218.** 28. **216.** PRECAUTION.

219. Divide the cross as indicated in the first figure and rearrange as shown in the latter.





224. Subtract the smallest number from each of the others. The G. C. D. of the differences is the required divisor. *Answer*, 2.

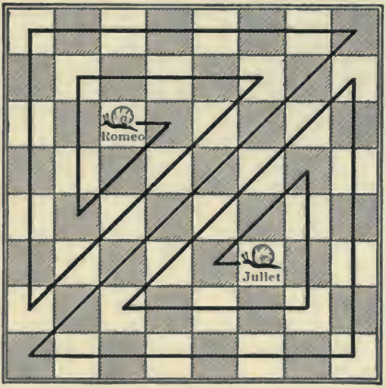
226. The number of shoes equals the number of persons.

227. 27.3083+ inches.

228. There would be no difference in the weight when the bird perched or flew. The air which supports the bird rests on the bottom of the cage. If the same cage had no top, the same would hold. If it had no bottom, there would still be no difference in weight. In this case the flight of the bird would tend to produce a vacuum just under the top, and the air above the cage would press downward with a force equal to the weight of the bird. If both top and bottom were removed, there would be a difference equal to the weight of the bird.

— From "School Science and Mathematics."

232.

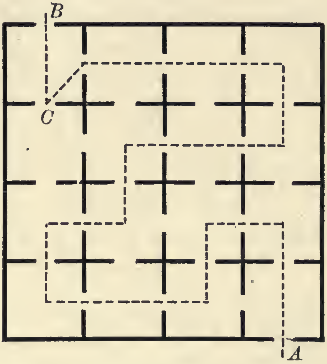


233. $2\frac{23278}{40831}$ inches and $\frac{11663}{40831}$ inches.

$(2\frac{23278}{40831})^3 + (\frac{11663}{40831})^3 = 17.$

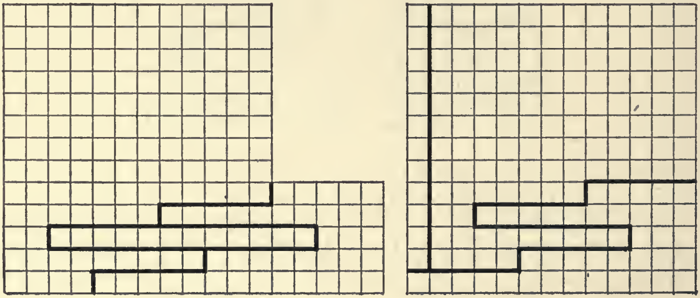
234. Similar solids are to each other as the cubes of corresponding lengths. Therefore the volumes of the balls are to each other as 1^3 is to 2^3 or as 1 to 8. By adding 1 to 8 we get 9. 9 is the sum of these two perfect cubes. We must now find two other numbers whose cubes added together make 9. These numbers must be fractional. They are $\frac{415280564497}{348671682660}$ feet and $\frac{676702467503}{348671682660}$ feet.

235. Yes. By 2,071,723 and 5,363,222,357.

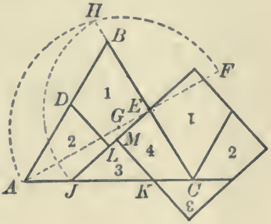


236. I entered the room *C* because I put my foot and part of my body in it, and I did not enter the other room twice, because after once going in I never left it until I made my exit at *B*. This is the only possible solution.

237.



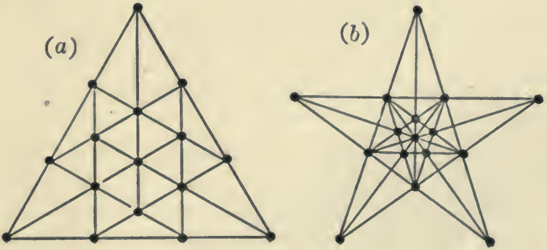
238. Bisect AB at D and BC at E; produce AE to F making EF equal to EB; bisect AF at G and describe the arc AHF; produce EB to H, and EH is the length of the side of the required square; from E with distance EH, describe the arc HJ and make JK equal to BE; now from the points D and K drop perpendiculars on EJ at L and M. If you have done this accurately, you will now have the required directions for the cuts.



241. 24. Reduce the length of the block by half an inch. The small block constitutes the waste. Cut the other piece into three pieces each $1\frac{1}{4}$ inches thick. Each of these may then be cut into eight blocks.

242. There are eleven times in twelve hours when the hour hand is exactly twenty minute spaces ahead of the minute hand. If we start at four o'clock and keep on adding 1 hour 5 minutes $27\frac{3}{11}$ seconds, we shall get all these eleven times, the last being 2 hours, 54 minutes, $32\frac{8}{11}$ seconds past twelve. Another addition brings us back to four o'clock, but at this time the second hand is nearly twenty-two minute spaces behind the minute hand, and if we examine all our eleven times, we shall find that only in one case is the second hand the required distance. This time is 54 minutes, $32\frac{8}{11}$ seconds past 2.

243.



244. 1—2—3—4—5; 1—2—4—5—3; 1—3—2—5—4; 1—3—4—2—5; 1—4—2—3—5; 1—4—3—5—2.

245. Let A, B, C, D, E, F, and G represent the seven men. The way of arranging them is as follows: —

A B C D E F G
A C D B G E F
A D B C F G E
A G B F E C D
A F C E G D B
A E D G F B C
A C E B G F D
A D G C F E B
A B F D E G C
A E F D C G B
A G E B D F C
A F G C B E D
A E B F C D G
A G C E D B F
A F D G B C E

246. $3\frac{1}{2}$ ft.

247. The bag contained either 79, 160, 241, 322, or 403, etc.

248. Twenty-six transfers are necessary. Move the cars so as to reach the following positions: —

$$\frac{E\ 5\ 6\ 7\ 8}{1\ 2\ 3\ 4\qquad\quad} = 10 \text{ transfers}$$

$$\frac{E\ 5\ 6}{1\ 2\ 3\quad 8\ 7\quad 4} = \quad 2 \text{ transfers}$$

$$\frac{5\ 6}{E\ 3\ 1\ 2\quad 8 7\quad 4} = \quad 5 \text{ transfers}$$

$$\frac{E}{8\ 7\ 6\ 5\ 4\ 3\ 2\ 1} = \quad 9 \text{ transfers.}$$

250. If there were twelve ladies in all, there would be 132 kisses among the ladies alone, leaving twelve more to be exchanged with the curate — six to be given by him and six to be received. Therefore of the twelve ladies, six would be his sisters. Consequently, if twelve could do the work in four and a half months, six ladies would do it in nine months.

252. Only three revolutions are necessary.

Number the nests from 1 to 12 in the direction the person travels. Transfer the egg in nest No. 1 to nest No. 2, in No. 5 to nest No. 8, in No. 9 to No. 12, in No. 3 to No. 6, in No. 7 to No. 10, in No. 11 to No. 2, and complete the last revolution to nest No. 1.

This can also be done by transferring the egg in nest No. 4, to No. 7, in No. 8 to No. 11, in No. 12 to No. 3, in No. 2 to No. 5, in No. 6 to No. 9, in No. 10 to No. 1.

253. He divided the rope in half. He simply untwisted the strands and divided it into two ropes, each being of the original length of the rope. He then tied these two ropes together and had a rope almost twice as long as the original rope.

254. 26.029962661171957726998490768328505774732373764 7323555652999.

255. I reached the shore with little difficulty. I fastened one end of the trot line to the stern of the boat, and then while standing in the bow, gave the line a series of violent jerks thus propelling the boat forward.

256. C's age at A's birth + A's present age = A's present age + B's; then C's age at A's birth = B's present age. By the second condition, A's age $- 3 = \frac{3}{4}$ (B's $+ 4$), from which A's age $= \frac{3}{4}$ B's age $+ 6$ years. The difference between A's and B's present ages = B's age at birth of A. Therefore $\frac{1}{4}$ of B's present age $- 6$ years = B's age at A's birth, and $5\frac{1}{2}$ ($\frac{1}{4}$ B's age $- 6$ years) $= \frac{11}{8}$ of B's present age, from which $\frac{11}{8}$ B's age $- 33$ years = B's age, or 88 years. C's age at A's birth was also 88; B's, $88 \div 5\frac{1}{2}$, or 16 years. A's present age is $88 - 16 = 72$ years; B's 88, and C's $88 + 72 = 160$ years.

257. £2,567 18 *s.* $9\frac{3}{4}$ *d.*

SHORT METHODS

Business men everywhere complain that the schools teach neither accuracy nor rapidity in calculations. They claim that the pupils must learn facts and principles and have much practice in the application of principles; that because a boy can apply a principle to-day is no guarantee that he will have the same knowledge and ability tomorrow; that eternal vigilance is not only the price of liberty, but also the price of proficiency.

"The mechanic who is not skillful in the use of his tools will never rise above poor mediocrity; the pupils' arithmetical tools are figures, and unless he can handle these with facility and accuracy, he must ever remain a plodder, a waster of time, and a blunderer upon whose results none can depend."

We are living in a fast age, an age of steam and electricity, when results are attained by lightning methods.

ADDITION

There are no short cuts in addition; every figure in every column must be added to ascertain the amount. Nevertheless the time required to perform an operation in addition can be substantially shortened in the following ways:

1. By making plain, legible figures.
2. By placing units of a certain order immediately beneath units of a like order.
3. By omitting the "ands" and "ares."
4. By making combinations of 10.
5. By double column adding.

1.

Civil Service Method

When long columns are to be added, the following method will be found practical.

485
576
324
449
625
264
33
29
24
2723

To insure accuracy, add each column from top downwards as well as from bottom upwards.

2.

Two-Column Adding

OPERATION

24
36
21
83
62
53
49
328

Explanation. To add 2 columns at a time, begin with the number at the bottom and add the units of the number next above, and then add the tens, naming the totals only. Continue in this way until all the numbers are added. Thus, the given example would read 49, 52, 102, 104, 164, 167, 247, 248, 268, 274, 304, 308, 328.

3.

To add Three or More Columns

OPERATION

142
381
212
468
1203

Explanation. Three columns or more may be added at one time by extending the two-column method to include all the columns desired. Thus, 468, 470, 480, 680, 681, 761, 1061, 1063, 1103, 1203.

4.

A Jap Method of Adding

Illustrative Example. — Find the sum of 382, 498, 364, 899, 842, and 789.

OPERATION
3 8 2
7.7.0
1 1.4 4
2 0.3.3
2 9 8 5
2.6.6.4 *Ans.*

EXPLANATION. — Write 382 as read. To add 498, say $4 + 3 = 7$; $9 + 8 = 17$, write .7, the dot (.) shows that 1 ten is to be carried; $8 + 2 =$ .0.

To add 364, say $3 + 7. = 11$, the dot following the 7 increases its value 1 and is read 8.

Continuing this method, we obtain 2.6.6.4 as the result, which would be read 3774.

NOTE. — To be an adder of any consequence, one ought to be able to add at least one hundred figures per minute.

SUBTRACTION

There are three common methods of subtraction. In the following example, we may say,

(1) 6 from 15, 9; 2 from 3, 1; 4 from 13, 9;
(2) 6 from 15, 9; 3 from 4, 1; 4 from 13, 9;
(3) 6 and 9, 15; 2 and 1 and 1, 4; 4 and 9, 13.

$$\begin{array}{r} 1345 \\ \underline{426} \\ 919 \end{array}$$

Each of these methods is easily understood. The first is the simplest of explanation, and hence it is generally taught to children. The second is slightly more rapid than the first. But the third, familiar to all as the common method of "making change," is so much more rapid than either of the others that it is recommended to all computers. This method is called the "Addition Method."

MULTIPLICATION

The squares of all numbers up to 30 should be memorized. They become the basis of further knowledge of numbers. Thus:

$13 \times 13 = 169$
$14 \times 14 = 196$
$15 \times 15 = 225$
$16 \times 16 = 256$
$17 \times 17 = 289$
$18 \times 18 = 324$
$19 \times 19 = 361$
$20 \times 20 = 400$
$21 \times 21 = 441$
$22 \times 22 = 484$
$23 \times 23 = 529$
$24 \times 24 = 576$
$25 \times 25 = 625$
$26 \times 26 = 676$
$27 \times 27 = 729$
$28 \times 28 = 784$
$29 \times 29 = 841$
$30 \times 30 = 900$

1.

WHEN THE MULTIPLICAND AND MULTIPLIER ARE ALTERNATING NUMBERS

Alternating numbers are those having in their regular order a number between them; as 7 and 9; 19 and 21; 32 and 34.

Rule. — *Write the square of the intermediate number less one.*

Example. — $15 \times 17 = 16^2 - 1 = 256 - 1 = 255.$
$17 \times 19 = 18^2 - 1 = 324 - 1 = 323.$
$39 \times 41 = 40^2 - 1 = 1600 - 1 = 1599.$

NOTE. — The product of two numbers having three intermediate numbers between them is equal to the square of the central number less 4. Thus $9 \times 13 = 11^2 - 4 = 117.$

2. When the multiplier is a composite number.

Multiply 328 by 42.

OPERATION

$$\begin{array}{r} 328 \\ \underline{7} \\ 2296 \\ \underline{6} \\ 13776 \end{array} \quad \textit{Ans.}$$

EXPLANATION. — The factors of 42 are 7 and 6. We multiply 328 by 7, and this result by 6 and obtain 13,776.

3. When the right-hand figure of the multiplier is 1.

Multiply 23,425 by 41.

OPERATION

$$\begin{array}{l} 23425 \text{ by } 41 \\ \underline{93700} \\ 960425 \quad \textit{Ans.} \end{array}$$

EXPLANATION. — Multiply the units' figure of the multiplicand by the tens' figure of the multiplier and set the figure of the product obtained one place to the left of units' figure of the multiplicand. Continue in this manner until all the figures of the multiplicand have been multiplied by the figures of the multiplier, and add the product, or products, thus found to the multiplicand and the result will be the product desired.

4. When the multiplier is a unit of any order.

Rule. — *Annex as many ciphers to the multiplicand as there are ciphers in the multiplier.*

Thus $42 \times 10 = 420$; $21 \times 100 = 2100$, etc.

5. When the multiplier is 11.

Rule. — *Beginning with units, add each term of the multiplicand to the one preceding, carrying as in the regular rule.*

Multiply 1328 by 11.

OPERATION

$$\begin{array}{r} 1328 \\ \underline{11} \\ 14608 \end{array} \quad \textit{Ans.}$$

Explanation. — $0 + 8 = 8$ and we write 8 for the units' figure of the product; $8 + 2 = 10$, we write 0 for tens' place; $2+3=5$ and 1 carried $= 6$; we write 6; $3 + 1 = 4$, we write 4; $1+0=1$, we write 1 and the product is 14,608.

6. When the multiplier is 9, 99, or any number of 9's.

Rule. — *Annex to the multiplicand as many ciphers as the multiplier contains 9's, and subtract the multiplicand from the result.*

Thus $43561 \times 999 = 43{,}561{,}000 - 43{,}561 = 43{,}517{,}439$, *Ans.*

7. To multiply any two figures by 11.

Rule. — *Add the figures and place the result between them.*

Thus $42 \times 11 = 462$, $29 \times 11 = 319$, etc.

8. To multiply by any number which ends with 9.

Multiply 327 by 39.

OPERATION

$$\begin{array}{r} 327 \\ \underline{40} \\ 13080 \\ \underline{327} \\ 12753 \end{array} \quad \textit{Ans.}$$

Explanation. — The next number higher than 39 is 40. Multiplying the multiplicand by 40 produces a result of 13,080. The real multiplier is one less than 40, therefore by subtracting once the multiplicand from the result we get the desired product.

9. To multiply by 15, 150, and 1500.

Multiply 324 by 15.

OPERATION

$$\begin{array}{l} 3240 \\ \underline{1620} \\ 4860 \end{array} \quad \textit{Ans.}$$

Explanation. — Annex a cipher to the multiplicand, take one half of that number and add to it and you have the desired product.

To multiply by 150, annex two ciphers, and to multiply by 1500 annex three ciphers.

10. To multiply two numbers ending in 5.

Rule. — *To multiply two small numbers each ending in 5, such* *as 35 and 75, take the product of the left-hand figures (the 3 and 7), increased by half their sum, and prefix the result to 25.*

Thus

$$\begin{array}{r} 35 \\ \underline{75} \\ 2625,\ \textit{Ans.} \end{array} \qquad \begin{array}{l} 5 \times 5 = 25. \\ 3 \times 7 + \frac{1}{2}(3+7) = 26. \end{array}$$

11. To square any number of two digits.

Rule. — *Square the figure in units' place to obtain the figure in units' place of the answer and carry as in multiplication. Then take twice the product of the figures in units' and tens' place, plus the amount carried. To the part of the square thus far obtained prefix the square of the figure in tens' place plus the amount carried.*

Thus $(84)^2 = 7056.$

$4^2 = 16.$ Put down 6 and carry 1.

$2(8 \times 4) + 1 = 65.$ Put down 5 and carry 6.

$8^2 + 6 = 70.$ Prefix 70 to 56.

This also applies to numbers of more than two digits, though not so readily performed mentally.

12. To square a number ending in 5.

Rule. — *To square a number ending in 5, such as 85, take the product of 8 by the next higher figure (9) and annex 25 to the result.*

Thus $85^2 = 7225.$

13. To square any number consisting of 9's.

Rule. — *Write as many 9's less one as there are in the given number, an 8, as many ciphers as 9's, and a 1.*

Thus $999^2 = 998001.$

14. To multiply by complements. Complements are useful not only in addition and subtraction, but also in multiplication. When the complements are small and the numbers of which they are complements are large, there is a great advantage in this method.

Multiply 98 by 95.

OPERATION

98 complement 2
95 complement 5
9310 10

EXPLANATION. — The product of the complements gives the two right-hand figures, 10, and subtracting either complement from the other factor gives the other two figures, 93.

Multiply 198 by 192.

OPERATION

198 complement 2
192 complement 8
38016 16

EXPLANATION. — When the numbers to be multiplied are between one hundred and two hundred, the remainder found by subtracting either complement from the other number must be doubled.

NOTE. — If the numbers to be multiplied are between two hundred and three hundred, the remainder must be multiplied by three; between three hundred and four hundred by four; between four hundred and five hundred by five; and so on.

15. To multiply by excesses.

Rule. — *From the sum of the numbers subtract 100 or 1000, as required, and annex the product of the excesses.*

NOTE. — An excess is the amount greater than 100, 1000, etc.

Example. — $112 \times 103 = 11536.$
$112 + 03 = 115.$
To 115 annex 12×3, or $36 = 11536.$

Example. — $1009 \times 1007 = 1016063.$
$1009 + 007 = 1016.$
To 1016 annex $063 = 1016063.$

DIVISION

When the divisor is an aliquot part of some higher unit.

1. To divide by $2\frac{1}{2}$, multiply the dividend by 4 and point off one place.

2. To divide by 5, multiply the dividend by 2 and point off one place.

3. To divide by 10, point off one place.

4. To divide by $12\frac{1}{2}$, multiply the dividend by 8 and point off two places.

5. To divide by $16\frac{2}{3}$, multiply the dividend by 6 and point off two places.

6. To divide by 20, multiply the dividend by 5 and point off two places.

7. To divide by 25, multiply the dividend by 4 and point off two places.

8. To divide by $33\frac{1}{3}$, multiply the dividend by 3 and point off two places.

9. To divide by 50, multiply the dividend by 2 and point off two places.

10. To divide by $66\frac{2}{3}$, multiply the dividend by 3, point off two places, and divide by 2.

11. To divide by 100, point off two places.

12. To divide by 125, multiply the dividend by 8 and point off three places.

13. To divide by 200, multiply the dividend by 5 and point off three places.

14. To divide by 250, multiply the dividend by 4 and point off three places.

15. To divide by 500, multiply the dividend by 2 and point off three places.

16. To divide by 1000, point off three places.

FRACTIONS

1. To add two fractions which have 1 for their numerator.

Rule. — *Write the sum of the given denominators over the product of the given denominators.*

Thus $\frac{1}{4} + \frac{1}{5} = \frac{9}{20}$.

2. To subtract two fractions which have 1 for their numerator.

Rule. — *Write the difference of the given denominators over the product of the given denominators.*

Thus $\frac{1}{4} - \frac{1}{6} = \frac{2}{24} = \frac{1}{12}$.

3. To multiply two mixed numbers when the whole numbers are the same and the sum of the fractions is 1.

Rule. — *Multiply the whole number by the next highest whole number, and to the product thus obtained add the product of the fractions.*

Thus $9\frac{4}{5} \times 9\frac{1}{5} = 90\frac{4}{25}$.

4. To multiply two mixed numbers when the difference of the whole numbers is 1, and the sum of the fractions is 1.

Rule. — *Multiply the larger number increased by 1, by the smaller number; then square the fraction belonging to the larger number and subtract its square from 1. Add the whole number and the fraction and you have the desired product.*

Thus $5\frac{4}{5} \times 4\frac{1}{5} = 24\frac{9}{25}$.

5. To multiply two mixed numbers ending in $\frac{1}{2}$.

Rule. — *To the product of the whole numbers, add half their sum plus $\frac{1}{4}$. (If the sum be an odd number, call it one less, to make it even, and annex $\frac{3}{4}$.)*

Thus $8\frac{1}{2} \times 6\frac{1}{2} = 55\frac{1}{4}$, $5\frac{1}{2} \times 6\frac{1}{2} = 35\frac{3}{4}$, etc.

6. To square any number ending in one half.

Rule. — *Multiply the number by itself increased by unity, and annex $\frac{1}{4}$.*

7. To square any number ending in one fourth.

Rule. — *Multiply the number by itself increased by $\frac{1}{2}$, and annex $\frac{1}{16}$.*

8. To square any number ending in three fourths.

Rule. — *Multiply the number by itself increased by $1\frac{1}{2}$, and annex $\frac{9}{16}$.*

9. To square any number ending in one third.

Rule. — *Multiply the number by itself increased by* $\frac{2}{3}$, *and annex* $\frac{1}{9}$.

10. To square any number ending in two thirds.

Rule. — *Multiply the number by itself increased by* $1\frac{1}{3}$, *and annex* $\frac{1}{9}$.

11. To multiply two numbers ending with the same fraction.

Rule. — *To the product of the whole numbers, add that fraction of their sum, and the square of the fraction.*

Thus $15\frac{2}{7} \times 6\frac{2}{7} = 90 + 6 + \frac{4}{49} = 96\frac{4}{49}$.

12. To square any mixed number.

Rule. — *Multiply the whole number by itself increased by twice the fraction, and add the square of the fraction.*

INTEREST

1. The Thirty-six Per Cent Method.

Rule. — *Multiply the principal by the time in days, move the decimal point three places to the left, and divide:*

If at 1 % by 36.
If at 2 % by 18.
If at 3 % by 12.
If at 4 % by 9.
If at 5 % by 7.2.
If at 6 % by 6.
If at 7 % by 5.143.
If at 8 % by 4.5.
If at 9 % by 4.
If at 10 % by 3.6.
If at 11 % by 3.273.
If at 12 % by 3.

2. The Bankers' Sixty-day Method.

Rule. — (*a*) *Moving the decimal point in the principal three places to the left gives the interest at 6 % for 6 days.*

Moving the decimal point in the principal two places to the left gives the interest at 6 % for 60 days.

Moving the decimal point in the principal one place to the left gives the interest at 6 % for 600 days.

Writing the principal for the interest gives the interest at 6 % for 6000 days.

(*b*) *The interest for any other time or rate can easily be found by using convenient multiples or aliquot parts.*

Thus Interest on $ 36 for 6 days at 6 % = $ .036.
Interest on $ 36 for 60 days at 6 % = $ .36.
Interest on $ 36 for 600 days at 6 % = $ 3.60.
Interest on $ 36 for 6000 days at 6 % = $ 36.00.

Example. — Find the interest on $ 300 for 4 yr. 6 mo. 18 da. at 6 %.

OPERATION

$ 72.00 = interest for the number of years.
$ 9.00 = interest for the number of months.
$ .90 = interest for the number of days.
$ 81.90 = the required interest.

Explanation. — 6 % of $ 300 = $ 18, the interest for one year. 4 × $ 18 = $ 72, the interest for 4 years. $ 3 = the interest for 2 months. 3 × $ 3 = $ 9, the interest for 6 months. 3 × $ .30 = $ .90, the interest for 18 days.

3. The Six Per Cent Method.

Interest on $ 1 for 1 year = $ .06.
Interest on $ 1 for 1 month = $ .00$\frac{1}{2}$.
Interest on $ 1 for 1 day = $ .000$\frac{1}{6}$.

Rule. — *Multiply 6 cents by the number of years, $\frac{1}{2}$ a cent by the number of months, $\frac{1}{6}$ of a mill by the number of days, and multiply their sum by the principal.*

Example. — Find the interest on $ 400 at 6 % for 6 yr. 4 mo. 12 da.

OPERATION

$ .36 = interest on $ 1 for number of years.
.02 = interest on $ 1 for number of months.
.002 = interest on $ 1 for number of days.
$ .382 = interest on $ 1 for the given time.
400
$ 152.80 = the required interest.

4. The Cancellation Method.

(1) When the time is in years.

Formula:

$$\text{Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}.$$

(2) When the time is in months.

Formula:

$$\text{Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100 \times 12}.$$

(3) When the time is in days.

Formula:

$$\text{Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100 \times 360}.$$

$$\text{Exact Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100 \times 365}.$$

Example.—Find the interest on $600 at 12% for 1 year, 3 months, 12 days.

OPERATION

$$\frac{\overset{6}{\cancel{600}} \times \cancel{12} \times 462}{\cancel{100} \quad \underset{\underset{5}{\cancel{60}}}{\cancel{360}}} = \$92.40, \text{ interest.}$$

5. The New Cancellation Method.

Rule. — *Write the principal, time, and rate at the right of a vertical line; at the left of this line write a year in the same denomination in which the time is expressed. Cancel and reduce. The result will be the interest for the given time and rate.*

Example.—Find the interest on $1080 for 3 yr. 4 mo. 12 da. at 6%.

OPERATION

$$\begin{array}{r|r} & \$\ 90 \\ & \cancel{1080} \\ \cancel{12} & 40.4 \\ & .06 \\ \hline \multicolumn{2}{r}{\$218.16 = \text{interest.}} \end{array}$$

Example.—Find the interest on $540 for 2 yr. 4 mo. 12 da. at 10%.

OPERATION

$$\begin{array}{r|r} & 6 \\ & \$\ \cancel{540} \\ \cancel{4} & 213 \\ \cancel{360} & \cancel{852} \\ & .10 \\ \hline \multicolumn{2}{r}{\$127.80 = \text{interest.}} \end{array}$$

6. The Cancellation-Thirty-six Per Cent Method.

Formula:

$$\text{Interest} = \frac{.001 \text{ of Principal} \times \text{Number of Days} \times \text{Rate}}{36}.$$

This method is a combination of the Cancellation Method and Thirty-six Per Cent Method and should be very popular on account of its simplicity.

Example. — Find the interest on $5112 at 4 % for 100 days.

OPERATION

$$\frac{\overset{\$.568}{\cancel{5}.\cancel{1}\cancel{1}\cancel{2}} \times 100 \times \cancel{4}}{\underset{\cancel{9}}{\cancel{3}\cancel{6}}} = \$56.80, \text{ interest.}$$

7. The Twelve Per Cent Method.

To find the interest for 1 month on any principal at 12 %, simply remove the decimal point two places to the left in the principal; in other words, divide the principal by 100. This gives the interest for 1 month at 12 %.

Rule. — *Point off two places in the principal, and multiply by the time expressed in months and decimals, or fractions of a month.*

Example. — What is the interest on $185 at 12 % for 3 months, 15 days?

OPERATION

$1.85 = interest at 12 % for 1 month.

$3\frac{1}{2}$ = time in months.

6.47\frac{1}{2}$ = interest for $3\frac{1}{2}$ months, *Ans.*

APPROXIMATE RESULTS

In scientific investigations exact results are rarely possible, since the numbers used are obtained by observation or by experiments and are only approximate. There is a degree of accuracy beyond which it is impossible to go.

The student should always bear in mind that it is a waste of time to carry out results to a greater degree of accuracy than the data on which they are founded. Results beyond two or three decimal places are seldom desired in business.

1. Multiplication.

Rule. — I. *Write the terms of the multiplier in a reverse order, placing the units' term under that term of the multiplicand which is of the lowest order in the required product.*

II. *Multiply each term of the multiplicand by the multiplier, rejecting those terms that are on the right of the term used as a multiplier, increasing each partial product by as many units as would have been carried to it from the product of the rejected part of the multiplicand, and one more when the second term towards the right in the product of the rejected terms is 5 or more than 5; and place the right-hand terms of these partial products in the same column.*

III. *Add the partial products, and point off in the sum the required number of decimal places.*

Example.—Multiply 4.78567 by 3.14159, correct to four decimal places.

OPERATION

$$
\begin{array}{rl}
4.78567 & \\
\underline{95141.3} & \\
14.3570 & = 4.7856 \times 3 + .0002. \\
.4786 & = 4.785 \times .1 + .0001. \\
.1914 & = 4.78 \times .04 + .0002. \\
48 & = 4.7 \times .001 + .0001. \\
24 & = 4 \times .0005 + .0004. \\
\underline{4} & = 0 + .00009 + .0004. \\
15.0346 &
\end{array}
$$

2. Division.

Rule. — I. *Compare the divisor with the dividend to ascertain the number of terms in the quotient.*

II. *For the first contracted divisor, take as many terms of the divisor, beginning with the first significant term on the left, as there are terms in the quotient; and for each successive divisor, reject the right-hand term of the previous divisor, until all the terms of the divisor have been rejected.*

III. *In multiplying by the several terms of the quotient, carry from the rejected terms of the divisor as in contracted multiplication.*

Example. — Divide 35.765342 by 8.76347, correct to four decimal places.

OPERATION

$$\begin{array}{l}
8.76347)35.765342(4.0811 \\
\underline{35\ 0539} = 4 \times 87634 + 3 \\
\quad 7114 \\
\quad \underline{7010} = 8 \times 876 + 2 \\
\quad\ \ 104 \\
\quad\ \ \ \underline{88} = 1 \times 87 + 1 \\
\quad\ \ \ 16 \\
\quad\ \ \ \ \underline{9} = 1 \times 8 + 1 \\
\quad\ \ \ \ 7
\end{array}$$

3. Square Root.

Rule. — *Find, as usual, more than one-half the terms of the root, and then divide the last remainder by the last divisor, using the contracted method.*

Example. — Extract the square root of 10.

OPERATION		CONTRACTED METHOD	
	10(3.16227766+		10(3.16227766+
	9		9
61	100	61	100
	61		61
626	3900	626	3900
	3756		3756
6322	14400	6322	14400
	12644		12644
63242	175600	63242	175600
	126484		126484
632447	4911600		49116
	4427129		44269
6324547	48447100		4847
	44271829		4427
63245546	417527100		420
	379473276		379
632455526	3805382400		41
	3794733156		38

4. Cube Root.

Rule. — *Extract the cube root, as usual, until one more than half the terms required in the root have been found; then with* *the trial divisor and last remainder proceed, as in contracted division of decimals, to find the other terms of the root, dropping two figures instead of one from the divisor at each step, and one from each remainder.*

Example. — Extract the cube root of 2 to four decimal places.

OPERATION

$$\begin{array}{r|l}
 & 2.000000 \;\underline{|\,1.2599} \\
 & 1 \\
\hline
300 & 1000 \\
60 & \\
\underline{4} & \\
364 & \;\;\underline{728} \\
43200 & \;\;272000 \\
1800 & \\
\underline{25} & \\
45025 & \;\;\underline{225125}
\end{array}$$

$$\text{Next trial divisor, } 4\cancel{6}\cancel{8}\cancel{7}\cancel{5} \;|\; 4687\cancel{5} \text{ remainder.}$$

$$\begin{array}{l}
\underline{4219} = 9 \times 468 + 9 \times 75 \\
\;\;46\cancel{8} \\
\;\;\underline{42} = 4 \times 9 + 6 \\
\;\;\;\;4
\end{array}$$

5. Extraction of Any Root.

Rule. — *Obtain one less than half of the figures required in the root as the rule directs; then, instead of annexing ciphers and bringing down a period to the last numbers in the columns, leave the remainder in the right-hand column for a dividend; cut off the right-hand figure from the last number of the previous column, two right-hand figures from the last number in the column before that, and so on, always cutting off one more figure for every column to the left.*

With the number in the right-hand column and the one in the previous column, determine the next figure of the root, and use it as directed in the rule, recollecting that the figures cut off are not used except in carrying the tens they produce.

This process is continued until the required number of figures *is obtained, observing that when all the figures in the last number of any column are cut off, that column will be no longer used.*

Remark. — Add to the 1st column mentally; multiply and add to the next column in one operation: multiply and subtract from the right-hand column in like manner.

Example. — Extract the cube root of 44.6 to six decimals.

OPERATION

0	0	44.600(3.546323
3	9	17600
6	2700	1725000
90	3175	238136
95	367500	12182
100	371716	865
1050	37594~~8~~	111
1054	37659	
1058	37~~723~~	
~~1062~~		

Remark. — The trial divisors may be known by ending in two ciphers; the complete divisors stand just beneath them. After getting 3 figures of the root, contract the operation by last rule.

— From Ray's "Higher Arithmetic."

MARKING GOODS

To find the selling price of a single article at a certain per cent profit when the price per dozen and rate per cent gain are given.

Thus, to make 5 per cent, multiply the cost per dozen by $.08\frac{3}{4}$.

6 %	multiply by $.08\frac{5}{6}$	40 %	multiply by $.11\frac{2}{3}$
8 %	multiply by .09	45 %	multiply by $.12\frac{1}{12}$
10 %	multiply by $.09\frac{1}{6}$	50 %	multiply by $.12\frac{1}{2}$
$12\frac{1}{2}$ %	multiply by $.09\frac{3}{8}$	55 %	multiply by $.12\frac{11}{12}$
15 %	multiply by $.09\frac{7}{12}$	60 %	multiply by $.13\frac{1}{3}$
20 %	multiply by .10	65 %	multiply by $.13\frac{3}{4}$
25 %	multiply by $.10\frac{5}{12}$	$66\frac{2}{3}$ %	multiply by $.13\frac{8}{9}$
30 %	multiply by $.10\frac{5}{6}$	75 %	multiply by $.14\frac{7}{12}$
$33\frac{1}{3}$ %	multiply by $.11\frac{1}{9}$	80 %	multiply by .15
35 %	multiply by $.11\frac{1}{4}$	100 %	multiply by $.16\frac{2}{3}$

QUOTATIONS ON MATHEMATICS

"Mathematics, the queen of the sciences." — Gauss.

"Mathematics, the science of the ideal, becomes the means of investigating, understanding, and making known the world of the real." — White.

"Mathematics is the glory of the human mind." — Leibnitz.

"The two eyes of exact science are mathematics and logic." — De Morgan.

"Mathematics is the science which draws necessary conclusions from given premises." — Pierce.

"The advance and the perfecting of mathematics are closely joined to the prosperity of the nation." — Napoleon.

"Geometry is the perfection of logic, and excels in training the mind to logical habits of thinking. In this respect it is superior to the study of logic itself, for it is logic embodied in the science of tangible form." — Brooks.

"God geometrizes continually," was Plato's reply when questioned as to the occupation of the Deity.

"There is no royal road to geometry." — Euclid.

"Let no one who is unacquainted with geometry enter here," was the inscription over the entrance into the academy of Plato the philosopher.

"All scientific education which does not commence with mathematics is, of necessity, defective at its foundation." — Comte.

"A natural science is a science only in so far as it is mathematical." — Kant.

"Mathematics is the language of definiteness, the necessary vocabulary of those who know." — WHITE.

"The laws of nature are but the mathematical thoughts of God." — KEPLER.

"Mathematics is the most marvelous instrument created by the genius of man for the discovery of truth." — LAISANT.

"Euclid has done more to develop the logical faculty of the world than any book ever written. It has been the inspiring influence of scientific thought for ages, and is one of the cornerstones of modern civilization." — BROOKS.

"Mathematics is thinking God's thought after Him. When anything is understood, it is found to be susceptible of mathematical statement. The vocabulary of mathematics is the ultimate vocabulary of the material universe." — WHITE.

"Geometry is regarded as the most perfect model of a deductive science, and is the type and model of all science." — BROOKS' "Mental Science."

"I have always treated and considered puzzles from an educational standpoint, for the reason that they constitute a species of mental gymnastics which sharpen the wits, clear fog and cobwebs from the brain, and school the mind to concentrate properly. Comparatively but few people know how to think properly. As a school for mechanical ingenuity, for stirring up the gray matter in the brain, puzzle practice stands unique and alone." — SAM LOYD.

"Geometry not only gives mental power, but it is a test of mental power. The boy who cannot readily master his geometry will never attain to much in the domain of thought. He may have a fine poetic sense that will make a writer or an orator; but he can never reach any eminence in scientific thought or philosophic opinion. All the great geniuses in the realm of science, as far as known, had fine mathematical abilities. So valuable is geometry as a discipline that many lawyers and preachers review their geometry every year in order to keep the mind drilled to logical habits of thinking." — BROOKS' "Mental Science."

"Mathematics is the very embodiment of truth. No true devotee of mathematics can be dishonest, untruthful, unjust. Because, working ever with that which is true, how can one develop in himself that which is exactly opposite? It would be as though one who was always doing acts of kindness should develop a mean and groveling disposition. Mathematics, therefore, has ethical value as well as educational value. Its practical value is seen about us every day. To do away with every one of the many conveniences of this present civilization in which some mathematical principle is applied, would be to turn the finger of time back over the dial of the ages to the time when man dwelt in caves and crouched over the bodies of wild beasts." — B. F. FINKEL.

"As the drill will not penetrate the granite unless kept to the work hour after hour, so the mind will not penetrate the secrets of mathematics unless held long and vigorously to the work. As the sun's rays burn only when concentrated, so the mind achieves mastery in mathematics, and indeed in every branch of knowledge, only when its possessor hurls all his forces upon it. Mathematics, like all the other sciences, opens its door to those only who knock long and hard. No more damaging evidence can be adduced to prove the weakness of character than for one to have aversion to mathematics; for whether one wishes so or not, it is nevertheless true, that to have aversion for mathematics means to have aversion to accurate, painstaking, and persistent hard study, and to have aversion to hard study is to fail to secure a liberal education, and thus fail to compete in that fierce and vigorous struggle for the highest and the truest and the best in life which only the strong can hope to secure." — B. F. FINKEL.

"Mathematics develops step by step, but its progress is steady and certain amid the continual fluctuations and mistakes of the human mind. Clearness is its attribute, it combines disconnected facts and discovers the secret bond that unites them. When air and light and the vibratory phenomena of electricity and magnetism seem to elude us, when bodies are removed from us into the infinitude of space, when man wishes to behold the drama of the heavens that has been enacted centuries ago, when he wants to investigate the effects of gravity and heat in the deep, impenetrable interior of our earth, then he calls to his aid the help of mathematical analysis. Mathematics renders palpable the most intangible things, it binds the most fleeting phenomena, it calls down the bodies from the infinitude of the heavens and opens up to us the interior of the earth. It seems a power of the human mind conferred upon us for the purpose of recompensing us for the imperfection of our senses and the shortness of our lives. Nay, what is still more wonderful, in the study of the most diverse phenomena it pursues one and the same method, it explains them all in the same language, as if it were to bear witness to the unity and simplicity of the plan of the universe." — FOURIER.

"The practical applications of mathematics have in all ages redounded to the highest happiness of the human race. It rears magnificent temples and edifices, it bridges our streams and rivers, it sends the railroad car with the speed of the wind across the continent; it builds beautiful ships that sail on every sea; it has constructed telegraph and telephone lines and made a messenger of something known to mathematics alone that bears messages of love and peace around the globe; and by these marvelous achievements, it has bound all the nations of the earth in one common brotherhood of man."

— B. F. FINKEL.

"Mathematics is the indispensible instrument of all physical research." — BERTHELOT.

"It is in mathematics we ought to learn the general method always followed by the human mind in its positive researches."
— COMTE.

"All my physics is nothing else than geometry."
— DESCARTES.

"If the Greeks had not cultivated conic sections, Kepler could not have superseded Ptolemy." — WHEWELL.

"There is nothing so prolific in utilities as abstractions."
— FARADAY.

"I am sure that no subject loses more than mathematics by any attempt to dissociate it from its history." — GLAISHER.

"The history of mathematics is one of the large windows through which the philosophic eye looks into past ages and traces the line of intellectual development." — CAJORI.

"If we compare a mathematical problem with a huge rock, into the interior of which we desire to penetrate, then the work of the Greek mathematicians appears to us like that of a vigorous stonecutter who, with chisel and hammer, begins with indefatigable perseverance, from without, to crumble the rock slowly into fragments; the modern mathematician appears like an excellent miner who first bores through the rock some few passages, from which he then bursts it into pieces with one powerful blast, and brings to light the treasures within." — HANKEL.

"The world of ideas which mathematics discloses or illuminates, the contemplation of divine beauty and order which it induces, the harmonious connection of its parts, the infinite hierarchy and absolute evidence of truths with which mathematical science is concerned, these, and such like, are the surest grounds of its title to human regard." — SYLVESTER.

"I often find the conviction forced upon me that the increase of mathematical knowledge is a necessary condition for the advancement of science, and if so, a no less necessary condition for the improvement of mankind. I could not augur well for the enduring intellectual strength of any nation of men, whose education was not based on a solid foundation of mathematical learning and whose scientific conception, or in other words, whose notions of the world and of things in it, were not braced and girt together with a strong framework of mathematical reasoning." — H. J. STEPHEN SMITH.

"If the eternal and inviolable correctness of its truths lends to mathematical research, and therefore also to mathematical knowledge, a *conservative* character on the other hand, by the continuous outgrowth of new truths and methods from the old, *progressiveness* is also one of its characteristics. In marvelous profusion old knowledge is augmented by new, which has the old as its necessary condition, and, therefore, could not have arisen had not the old preceded it. The indestructibility of the edifice of mathematics renders it possible that the work can be carried to ever loftier and loftier heights without fear that the highest stories shall be less solid and safe than the foundations, which are the axioms, or the lower stories, which are the elementary propositions. But it is necessary for this that all the stones should be *properly fitted together*, and it would be idle labor to attempt to lay a stone that belonged above in a place below." — SCHUBERT.

"As the sun eclipses the stars by his brilliancy, so the man of knowledge will eclipse the fame of others in assemblies of the people if he proposes algebraic problems, and still more if he solves them." — BRAHMAGUPTA.

"Mathematical reasoning may be employed in the inductive sciences; indeed some of their greatest achievements have been obtained through mathematics. By it Newton demonstrated the truth of the theory of gravitation; by it Leverrier discovered a new planet in the heavens; by it the exact time of an eclipse of the sun or moon is predicted centuries before it comes to pass. Mathematics is the instrument by which the engineer tunnels our mountains, bridges our rivers, constructs our aqueducts, erects our factories and makes them musical with the busy hum of spindles. Take away the results of the reasoning of mathematics, and there would go with it nearly all the material achievements which give convenience and glory to modern civilization." — Brooks' "Mental Science and Culture."

"The science of geometry came from the Greek mind almost as perfect as Minerva from the head of Jove. Beginning with definite ideas and self-evident truths, it traces its way, by the processes of deduction, to the profoundest theorem. For clearness of thought, closeness of reasoning, and exactness of truths, it is a model of excellence and beauty. It stands as a type of all that is best in the classical culture of the thoughtful mind of Greece. Geometry is the perfection of logic; Euclid is as classic as Homer." — Brooks' "Philosophy of Arithmetic."

"Only a limited number of people are capable of appreciating the beauties of this oldest of all sciences." — Locke.

"The value of mathematical instruction as a preparation for those more difficult investigations consists in the applicability, not of its doctrines, but of its methods. Mathematics will ever remain the past-perfect type of the deductive method in general; and the applications of mathematics to the simpler branches of physics furnish the only school in which philosophers can effectually learn the most difficult and important portion of their art, the employment of the laws of simpler phenomena for explaining and predicting those of the more complex. These grounds are quite sufficient for deeming mathematical training an indispensable basis of real scientific education, and regarding, with Plato, one who is ἀγεωμέτρητος, as wanting in one of the most essential qualifications for the successful cultivation of the higher branches of philosophy." — From J. S. Mill's "Systems of Logic."

"Hold nothing as certain save what can be demonstrated." — NEWTON.

"To measure is to know." — KEPLER.

"It may seem strange that geometry is unable to define the terms which it uses most frequently, since it defines neither movement, nor number, nor space — the three things with which it is chiefly concerned. But we shall not be surprised if we stop to consider that this admirable science concerns only the most simple things, and the very quality that renders these things worthy of study renders them incapable of being defined. Thus the very lack of definition is rather an evidence of perfection than a defect, since it comes not from the obscurity of the terms, but from the fact that they are so very well known." — PASCAL.

"The method of making no mistake is sought by every one. The logicians profess to show the way, but the geometers alone ever reach it, and aside from their science there is no genuine demonstration." — PASCAL.

"We may look upon geometry as a practical logic, for the truths which it studies, being the most simple and most clearly understood of all truths, are on this account the most susceptible of ready application in reasoning." — D'ALEMBERT.

"Without mathematics no one can fathom the depths of philosophy. Without philosophy no one can fathom the depths of mathematics. Without the two no one can fathom the depths of anything." — BORDAS DEMOULIN.

"The taste for exactness, the impossibility of contenting one's self with vague notions or of leaning upon mere hypotheses, the necessity for perceiving clearly the connection between certain propositions and the object in view, — these are the most precious fruits of the study of mathematics." — LACROIX.

"God is a circle of which the center is everywhere and the circumference nowhere." — RABELAIS.

"The sailor whom an exact observation of longitude saves from shipwreck owes his life to a theory developed two thousand years ago by men who had in mind merely the speculations of abstract geometry." — CONDORCET.

"The statement that a given individual has received a sound geometrical training implies that he has segregated from the whole of his sense impressions a certain set of these impressions, that he has then eliminated from their consideration all irrelevant impressions (in other words, acquired a subjective command of these impressions), that he has developed on the basis of these impressions an ordered and continuous system of logical deduction, and finally that he is capable of expressing the nature of these impressions and his deductions therefrom in terms simple and free from ambiguity. Now the slightest consideration will convince any one not already conversant with the idea, that the same sequence of mental processes underlies the whole career of any individual in any walk of life if only he is not concerned entirely with manual labor; consequently a full training in the performance of such sequences must be regarded as forming an essential part of any education worthy of the name. Moreover, the full appreciation of such processes has a higher value than is contained in the mental training involved, great though this be, for it induces an appreciation of intellectual unity and beauty which plays for the mind that part which the appreciation of schemes of shape and color plays for the artistic faculties; or, again, that part which the appreciation of a body of religious doctrine plays for the ethical aspirations. Now geometry is not the sole possible basis for inculcating this appreciation. Logic is an alternative for adults, provided that the individual is possessed of sufficient wide, though rough, experience on which to base his reasoning. Geometry is, however, highly desirable in that the objective bases are so simple and precise that they can be grasped at an early age, that the amount of training for the imagination is very large, that the deductive processes are not beyond the scope of ordinary boys, and finally that it affords a better basis for exercise in the art of simple and exact expression than any other possible subject of a school course." — CARSON.

"Geometry is a mountain. Vigor is needed for its ascent. The views all along the paths are magnificent. The effort of climbing is stimulating. A guide who points out the beauties, the grandeur, and the special places of interest, commands the admiration of his group of pilgrims." — DAVID EUGENE SMITH.

"If mathematical heights are hard to climb, the fundamental principles lie at every threshold, and this fact allows them to be comprehended by that common sense which Descartes declared was 'apportioned equally among all men.'" — COLLET.

"The wonderful progress made in every phase of life during the last hundred years has been possible only through the increasing use of symbols. To-day, only the common laborer works entirely with the actual things. Those who occupy more remunerative positions in the business world work very largely with symbols, and in the professional world the possession of and ability to use a set of symbols is a prerequisite of even moderate success. The work of a man's hands remains after the worker has gone, but the products of mental labor are lost unless they are preserved to the world through some symbolic medium. It may be said without fear of successful contradiction that the language of mathematics is the most widely used of any symbolism. The man who has command of it possesses a clear, concise, and universal language. Fallacies in reasoning and discrepancies in conclusions are easily detected when ideas are expressed in this language. The most abstruse problem is immediately clarified when translated into mathematics. To quote from M. Berthelot, 'Mathematics excites to a high degree the conceptions of signs and symbols — necessary instruments to extend the power and reach of the human mind by summarizing. Mathematics is the indispensable instrument of all physical research.' But not only physical but all scientific research must avail itself of this same instrument. Indeed, so completely is nature mathematical that to him who would know nature there is no recourse but to be conversant with the language of mathematics." — CARPENTER.

No less an astronomer than J. Herschel has said of astronomy: "Admission to its sanctuary and to the privileges and feelings of a votary is only to be gained by one means — sound and sufficient knowledge of mathematics, the great instrument of all exact inquiry, without which no man can ever make such advances in this or any other of the higher departments of science as can entitle him to form an independent opinion on any subject of discussion within their range."

"It is only through mathematics that we can thoroughly understand what true science is. Here alone can we find in the highest degree simplicity and severity of scientific law, and such abstraction as the human mind can attain. Any scientific education setting forth from any other point is faulty in its basis." — COMTE.

"The enemies of geometry, those who know it only imperfectly, look upon the theoretical problems, which constitute the most difficult part of the subject, as mental games which consume time and energy that might better be employed in other ways. Such a belief is false, and it would block the progress of science if it were credible. But aside from the fact that the speculative problems, which at first sight seem barren, can often be applied to useful purposes, they always stand as among the best means to develop and to express all the forces of the human intelligence." — ABBÉ BOSSUT.

"We study music because music gives us pleasure, not necessarily our own music, but good music, whether ours, or, as is more probable, that of others. We study literature because we derive pleasure from books; the better the book, the more subtle and lasting the pleasure. We study art because we receive pleasure from the great works of the masters, and probably we appreciate them the more because we have dabbled a little in pigments or in clay. We do not expect to be composers, or poets, or sculptors, but we wish to appreciate music and letters and the fine arts, and to derive pleasure from them and to be uplifted by them. At any rate these are the nobler reasons for their study.

"So it is with geometry. We study it because we derive pleasure from contact with a great and an ancient body of learning that has occupied the attention of master minds during the thousands of years in which it has been perfected, and we are uplifted by it. To deny that our pupils derive this pleasure from the study is to confess ourselves poor teachers, for most pupils do have positive enjoyment in the pursuit of geometry, in spite of the tradition that leads them to proclaim a general dislike for all study. This enjoyment is partly that of the game, — the playing of a game that can always be won, but that cannot be won too easily. It is partly that of the æsthetic, the pleasure of symmetry of form, the delight of fitting things together. But probably it lies chiefly in the mental uplift that geometry brings, the contact with absolute truth, and the approach that one makes to the Infinite. We are not quite sure of any one thing in biology; our knowledge of geology is relatively very slight, and the economic laws of society are uncertain to every one except some individual who attempts to set them forth; but before the world was fashioned the square on the hypotenuse was equal to the sum of the squares on the other two sides of a right triangle, and it will be so after this world is dead; and the inhabitant of Mars, if he exists, probably knows its truth as we know it. The uplift of this contact with absolute truth, with truth eternal, gives pleasure to humanity to a greater or less degree, depending upon the mental equipment of the particular individual; but it probably gives an appreciable amount of pleasure to every student of geometry who has a teacher worthy of the name." — From "The Teaching of Geometry," by DAVID EUGENE SMITH.

Mathematics has not only commercial value, but also educational, rhetorical, and ethical value. No other science offers such a rich opportunity for original investigation and discovery. While it should be studied because of its practical worth, which can be seen about us every day, the primary object in its study should be to obtain mental power, to sharpen and strengthen the powers of thought, to give penetrating power to the mind which enables it to pierce a subject to its core and discover its elements; to develop the power to express one's thoughts in a forcible and logical manner; to develop the memory and the imagination; to cultivate a taste for neatness and a love for the good, the beautiful, and the true; and to become more like the greatest of mathematicians, the Mathematician of the Universe.

MENSURATION

Mensuration is that branch of mathematics which treats of the measurement of geometrical magnitudes.

ANNULUS, OR CIRCULAR RING

An annulus is the figure included between two concentric circumferences.

(1) To find the area of an annulus.

Rule. — *Multiply the sum of the two radii by their difference, and the product by* π.

Formula. — $A = (r_1 + r_2)(r_1 - r_2)\pi$.

(2) To find the area of a sector of an annulus.

Rule. — *Multiply the sum of the bounding arcs by half the difference of their radii.*

BELTS

Length of belts.

(*a*) For a crossed belt,

$$L = 2\sqrt{c^2 - (r_1 - r_2)^2} + (r_1 - r_2)\left(\pi - 2\sin^{-1}\frac{r_1 + r_2}{c}\right),$$

(*b*) For an uncrossed belt,

$$L = 2\sqrt{\{c^2 - (r_1 - r_2)^2\}} + \pi(r_1 + r_2) + 2(r_1 - r_2)\sin^{-1}\frac{r_1 - r_2}{c},$$

where r_1 is the greater radius and r_2 the less, and c the distance between the parallel axes.

Bins, Cisterns, Etc.

(1) To find the exact capacity of a bin in bushels.

Rule. — *Divide the contents in cubic feet by .83, or (1728 ÷ 2150.42); the quotient will represent the number of bushels of grain, etc. Four fifths of this number of bushels is the number of bushels of coal, apples, potatoes, etc., that the bin will hold.*

(2) To find the approximate capacity of a bin in bushels.

Rule. — *Any number of cubic feet diminished by ⅕ will represent an equivalent number of bushels.*

(3) To find the contents of a cistern, vessel, or space in gallons.

Rule. — *Divide the contents in cubic inches by 231 for liquid gallons, or by 268.8 for dry gallons.*

Brick and Stone Work

Stonework is commonly estimated by the perch; brickwork by the thousand bricks.

(1) In estimating the work of laying stone, take the entire outside length in feet, thus measuring the corners twice, times the height in feet, times the thickness in feet, and divide by 24¾, to obtain the number of perches. No allowance is to be made for openings in the walls unless specified in a written contract.

(2) In estimating the material in stonework, deduct for all openings and divide the exact number of cubic feet of wall by 24¾, to obtain the number of perches of material.

To obtain the number of perches of stone, deduct ⅛ for mortar and filling.

(3) In estimating the work of laying common bricks (common bricks are 8 inches × 4 inches × 2 inches and 22 are assumed to build 1 cubic foot), take the entire outside length in feet, thus measuring the corners twice, times the height in feet, times the thickness in feet, and multiply by .022, to obtain the

number of thousand bricks. No allowance is to be made for openings in the walls unless specified in a written contract.

(4) In estimating the material in brickwork, deduct for all openings and multiply the exact number of cubic feet of wall by 22, to obtain the number of brick required.

CARPETING

Carpets are usually either 1 yard or $\frac{3}{4}$ yard in width.

The amount of carpet that must be bought for a room depends upon the length and number of strips, and the waste in matching the patterns.

(1) To obtain the number of strips.

A fraction of a strip cannot be bought. Thus, if the number of strips is found to be $6\frac{1}{9}$, make it 7.

(*a*) When laid lengthwise. — Divide the width of the room in yards by the width of the carpet in yards.

(*b*) When laid crosswise. — Divide the length of the room in yards by the width of the carpet in yards.

(2) To obtain the number of yards of carpet needed to carpet a room.

Rule. — *Multiply the length of a strip in yards (+ the fraction of a yard allowed for waste, when considered) by the number of strips.*

CASKS AND BARRELS

To find the contents in gallons.

Rule. — *Add to the head diameter (inside) two thirds of the difference between the head and bung diameters; but if the staves are only slightly curved, add six tenths of this difference; this gives the mean diameter; express it in inches, square it, multiply it by the length in inches and this product by .0034: the product will be the contents in liquid gallons.*

CIRCLE

A circle is a portion of a plane bounded by a curved line every point of which is equally distant from a point within called the center.

(1) Formulæ. —

Area $= \pi r^2$, or $\frac{1}{4}\pi d^2$.
Area $= d^2 \times .7854$, or circumference$^2 \times .07958$.
Circumference $=$ diameter $\times$ 3.1416.
Circumference $=$ radius $\times$ 6.2832.
Diameter $=$ circumference $\times$ .31831.
Diameter $=$ circumference $\div \pi$.
Radius $=$ circumference $\times$ .159155.
Radius $= .56419 \times \sqrt{\text{area}}$.
Side of inscribed square $= d \times .707107$.
Side of inscribed square $=$ circumference $\times$ .22508.
Area of inscribed square $= \frac{1}{2} d^2$.
Side of an equal square $=$ circumference $\times$ .282.
Area of an equal square $= \frac{8}{9} d^2$.
Side of inscribed equilateral triangle $= d \times .86$.

(2) Given the area inclosed by three equal circles, to find the diameter of a circle that will just inclose the three equal circles.

Rule. — *Divide the given area by .03473265, extract the square root of the quotient, and multiply by 2, and the result will be the diameter required.*

Formula. — Diameter $= 2\sqrt{\dfrac{\text{area}}{.03473265}}$.

(3) To find the diameter of the three largest equal circles that can be inscribed in a circle of a given diameter.

Rule. — *Multiply the given diameter by .4641, or divide by 2.1557, and the result will be the required diameter.*

Formula. — $d = .4641 \times D$, or $\dfrac{D}{2.1557}$.

(4) Given the radius a, b, c, of the three circles tangent to each other, to find the radius of a circle tangent to the three circles.

Formula. — r or $r' = \dfrac{abc}{2\sqrt{[abc(a+b+c)]} \mp (ab+ac+bc)}$, the minus sign giving the radius of a tangent circle circumscribing the three given circles, and the plus sign giving the radius of a tangent circle inclosed by the three given circles.

— From "The School Visitor."

(5) Given the chord of an arc and the radius of the circle, to find the chord of half the arc.

Formula. — $k = \sqrt{2r^2 - r\sqrt{4r^2 - c^2}}$, where $r =$ radius and $c =$ the given chord.

(6) Given the chord of an arc and the radius of the circle, to find the height of the arc.

Rule. — *From the radius, subtract the square root of the difference of the squares of the radius and half the chord.*

Formula. — $h = r - \sqrt{r^2 - \frac{1}{4}c^2}$, where $r =$ radius and $c =$ the given chord.

(7) Given the height of an arc and a chord of half the arc, to find the diameter of the circle.

Rule. — *Divide the square of the chord of half the arc by the height of the chord.*

Formula. — $d = c^2 \div h$, where $c =$ chord of half the arc and $h =$ height.

(8) Given a chord and height of the arc, to find the chord of half the arc.

Rule. — *Extract the square root of the sum of the squares of the height of the arc and half the chord.*

Formula. — $k = \sqrt{h^2 + \frac{1}{4}c^2}$, where $h =$ height and $c =$ the given chord.

(9) Given the radius of a circle and a side of an inscribed polygon, to find the side of a similar circumscribed polygon.

Formula. — $s' = \frac{2\,sr}{\sqrt{4\,r^2 - s^2}}$, where s' = the side required and s = the side of the inscribed polygon.

CONE

A cone is a solid bounded by a conical surface and a plane.

(1) To find the lateral area of a right circular cone.

Rule. — *Multiply the circumference of its base by half the slant height.*

Formula. — Lateral area $= \pi rh$, where r = the radius of the base and h = the slant height.

(2) To find the volume of any cone.

Rule. — *Multiply the base by one third the altitude.*

Formula. — $V = \frac{1}{3}\,aB$.

Formula, when base is a circle. — $V = \frac{1}{3}\,ar^2\pi$, where a = altitude, B = base, and r = radius of the base.

CRESCENT

A crescent is a portion of a plane included between the corresponding arcs of two intersecting circles, and is the difference between two segments having a common chord, and on the same side of it.

CUBE OR HEXAHEDRON

Diagonal $= \sqrt{3 \times \text{edge}^2}$, or $\sqrt{\text{area} \div 2}$.
Diagonal $=$ edge $\times$ 1.7320508.
Surface $= 6 \times \text{edge}^2$, or $2 \times \text{diagonal}^2$.
Volume $= \text{edge}^3$.

CYCLOID

A cycloid is the curve generated by a point in the circumference of a circle which rolls on a straight line.

(1) To find the length of a cycloid.

Rule. — *Multiply the diameter of the generating circle by 4.*

(2) To find the area of a cycloid.

Rule. — *Multiply the area of the generating circle by 3.*

(3) To find the surface generated by the revolution of a cycloid about its base.

Rule. — *Multiply the area of the generating circle by* $\frac{64}{3}$.

(4) To find the volume of the solid formed by revolving the cycloid about its base.

Rule. — *Multiply the cube of the radius of the generating circle by* $5\pi^2$.

(5) To find the surface generated by revolving the cycloid about its axis.

Rule. — *Multiply eight times the area of the generating circle by* π *minus* $\frac{4}{3}$.

(6) To find the volume of the solid formed by revolving the cycloid about its axis.

Rule. — *Multiply* $\frac{3}{4}$ *of the volume of a sphere whose radius is that of the generating circle by* $\frac{3}{2}\pi^2 - \frac{8}{3}$.

(7) To find the surface formed by revolving the cycloid about a tangent at the vertex.

Rule. — *Multiply the area of the generating circle by* $\frac{32}{3}$.

(8) To find the volume formed by revolving a cycloid about a tangent at the vertex.

Rule. — *Multiply the cube of the radius of the generating circle by* $7\pi^2$.

CYLINDER

A cylinder is a solid bounded by a cylindric surface and two parallel planes.

(1) To find the lateral area of a right circular cylinder.

Rule. — *Multiply its length by the circumference of its base.*

(2) To find the volume of any cylinder.

Rule. — *Multiply the altitude of the cylinder by the area of its base.*

Formula. — $V = a \times B$.
Formula when base is a circle. — $V = a\pi r^2$.

(3) To find the surface common to two equal circular cylinders whose axes intersect at right angles.

Rule. — *Multiply the square of the radius of the intersecting cylinders by 16.*

(4) To find the volume common to two equal circular cylinders whose axes intersect at right angles.

Rule. — *Multiply the cube of the radius of the intersecting cylinders by* $5\frac{1}{3}$.

(5) To find the length of the maximum cylinder inscribed in a cube, the axis of the cylinder coinciding with the diagonal of the cube.

Formula. — Length $= \frac{1}{3} a\sqrt{3}$, where a is the edge of the cube.

(6) To find the volume of the maximum cylinder inscribed in a cube, the axis of the cylinder coinciding with the diagonal of the cube.

Formula. — $V = \frac{1}{18} \pi a^3 \sqrt{3}$, where a is the edge of the cube.

Density of a Body

The density of any substance is the number of times the weight of the substance contains the weight of an equal bulk of water.

To find the density of a body.

Rule. — *Divide the weight in grams by the bulk in cubic centimeters.*

DODECAEDRON

A dodecaedron is a polyedron of twelve faces.

(1) To find the area of a regular dodecaedron.

Rule. — *Multiply the square of an edge by 20.64573.*

(2) To find the volume of a regular dodecaedron.

Rule. — *Multiply the cube of an edge by 7.66312.*

ELLIPSE

An ellipse is a plane curve of such a form that if from any point in it two straight lines be drawn to two given fixed points, the sum of these straight lines will always be the same.

(1) To find the circumference of an ellipse, the transverse and conjugate diameters being known.

Rule. — *Multiply the square root of half the sum of the squares of the two diameters by 3.141592.*

(2) To find the area of an ellipse, the transverse and conjugate diameters being given.

Rule. — *Multiply the product of the diameters by .785398.*

FRUSTUM OF A CONE OR PYRAMID

A frustum of a cone or pyramid is the portion included between the base and a parallel section.

(1) To find the lateral surface.

Rule. — *Multiply the sum of the perimeters, or circumferences, by one half the slant height.*

(2) To find the entire surface.

Rule. — *Add to the lateral surface the areas of both ends, or bases.*

(3) To find the volume of a frustum of a cone or pyramid.

Rule. — *To the sum of the areas of both bases add the square root of the product, and multiply this sum by one third of the altitude.*

GRAIN AND HAY

(1) To find the quantity of grain in a bin.

Rule. — *Multiply the contents in cubic feet by .83, and the result will be the contents in bushels.*

(2) To find the quantity of corn in a wagon bed or in a bin.

Rule. — *(1) For shelled corn, multiply the contents in cubic feet by .83, and the result will be the contents in bushels.* **Rule.** — *(2) For corn on the cob, deduct one half for cob.* **Rule.** — *(3) For corn not "shucked," deduct two thirds for cob and shuck.*

(3) To find the quantity of hay in a stack or rick.

Rule. — *Divide the contents in cubic feet by 550 for clover or by 450 for timothy; the quotient will be the number of tons.*

(4) In well-settled stacks 15 cubic yards make one ton.

(5) When hay is baled, 10 cubic yards make one ton.

HEXAEDRON

(See Cube.)

HYPERBOLA

A hyperbola is a section formed by passing a plane through a cone in a direction to make an angle at the base greater than that made by the slant height.

To find the area of a hyperbola, the transverse and conjugate axes and abscissa being given.

Rule. — *(1) To the product of the transverse diameter and abscissa add $\frac{5}{7}$ of the square of the abscissa, and multiply the square root of the sum by 21.*

(2) Add 4 times the square root of the product of the transverse diameter and abscissa to the product last found, and divide the sum by 75.

(3) Divide 4 times the product of the conjugate diameter and abscissa by the transverse diameter, and this last quotient multiplied by the former will give the area required, nearly.

Icosaedron

An icosaedron is a polyedron of twenty faces.

(1) To find the area of a regular icosaedron.

Rule. — *Multiply the square of an edge by 8.66025.*

(2) To find the volume of a regular icosaedron.

Rule. — *Multiply the cube of an edge by 2.18169.*

Irregular Polyedron

To find the volume of any irregular polyedron.

Rule. — *Cut the polyedron into prismatoids by passing parallel planes through all its summits.*

Irregular Solids

To find the volume of any irregular solid.

Rule. — *Immerse the solid in a vessel of water and determine the quantity of water displaced.*

Logs

(1) To find the side of the squared timber that can be sawed from a log.

Rule. — *Multiply the diameter of the smaller end by .707.*

(2) To find the number of board feet in the squared timber that can be sawed from a log.

Rule. — *Multiply together one half the length in feet, the diameter of the smaller end in feet, and the diameter of the smaller end in inches.*

Problem. — Find the side, and the number of board feet, in the squared timber that can be sawed from a log whose length is 16 feet, and diameter of the smallest end 15 inches.

Solution. — By (1) the side is 15 inches × .707, or 10.605 inches. By (2) the number of the board feet is $\frac{16}{2} \times \frac{15}{12} \times 15 = 150$, *Ans.*

LUMBER

When boards are 1 inch thick or less, they are estimated by the square foot of surface, the thickness not being considered.

Thus a board 10 feet long, 1 foot wide, and 1 inch (or less) thick contains 10 square feet.

Hence, to find the number of board feet in a plank.

Rule. — *Multiply the length in feet by the width in feet by the thickness in inches.*

NOTE. — The average width of a board that tapers uniformly is one half the sum of the end widths.

LUNE

A lune is that portion of a sphere comprised between two great semicircles.

To find the area of a lune.

Rule. — *Multiply its angle in radians by twice the square of the radius.*

OCTAEDRON

An octaedron is a polyedron of eight faces.

(1) To find the area of a regular octaedron.

Rule. — *Multiply the square of an edge by 3.4641.*

(2) To find the volume of a regular octaedron.

Rule. — *Multiply the cube of an edge by .4714.*

PAINTING AND PLASTERING

Painting and plastering are usually estimated by the square yard. The processes of calculating the cost of painting and plastering vary so much in different localities that it is impossible to lay down any rule. Usually some allowance is made for doors, windows, etc., but there is no fixed rule as to how much should be deducted. Sometimes one half the area of the openings is deducted.

PAPERING

Wall paper is sold only by the roll, and any part of a roll is considered a whole roll.

The amount of wall paper required to paper a room depends upon the area of the walls and ceiling and the waste in matching.

(1) American paper is commonly 18 inches wide, and has 8 yards in a single roll, and 16 yards in a double roll. Foreign papers vary in width and length to the roll.

(2) Wall paper is usually put up in double rolls, but the prices quoted are for single rolls.

(3) Borders and friezes are sold by the yard and vary in width.

(4) The area of a single roll is 36 square feet, and allowing for all waste in matching, etc., will cover 30 square feet of wall.

(5) There is no fixed rule as to how much should be deducted for doors and windows. Some dealers deduct the exact area of the openings, while others deduct an approximate area, allowing 20 square feet for each.

(6) The number of single rolls required for the ceiling and for the walls must be estimated separately.

(7) To obtain the number of single rolls required for the ceiling.

Rule. — *Divide its area in square feet by 30.*

(8) To obtain the number of single rolls required for the walls.

Rule. — *From the area of the walls in square feet deduct the area of the openings, and divide by 30.*

PARABOLA

A parabola is the locus of a point whose distance from a fixed point is always equal to its distance from a fixed straight line.

(1) To find the length of any arc of a parabola cut off by a double ordinate.

Rule. — *When the abscissa is less than half the ordinate: To the square of the ordinate add $\frac{4}{3}$ of the square of the abscissa, and twice the square root of the sum will be the length of the arc.*

(2) To find the area of the parabola, the base and height being given.

Rule. — *Multiply the base by the height, and $\frac{2}{3}$ of the product will be the area.*

(3) To find the area of a parabolic frustum, having given the double ordinates of its ends and the distance between them.

Rule. — *Divide the difference of the cubes of the two ends by the difference of their squares and multiply the quotient by $\frac{2}{3}$ of the altitude.*

PARALLELOGRAM

A parallelogram is a quadrilateral whose opposite sides are parallel.

To find the area of any parallelogram.

Rule. — *Multiply the base by the altitude.*

PARALLELOPIPED

A parallelopiped is a prism whose bases are parallelograms.

To find the volume of any parallelopiped.

Rule. — *Multiply its altitude by the area of its base.*

PRISM

A prism is a polyedron whose ends are equal and parallel polygons, and its sides parallelograms.

(1) To find the lateral area of a prism.

Rule. — *Multiply a lateral edge by the perimeter of a right section.*

(2) To find the volume of any prism.

Rule. — *Multiply the area of the base by its altitude.*

PRISMATOID

A prismatoid is a polyedron whose bases are any two polygons in parallel planes, and whose lateral forces are triangles determined by so joining the vertices of these bases that each lateral edge with the preceding forms a triangle with one side of either base.

(1) To find the volume of any prismatoid.

Rule. — *Add the areas of the two bases and four times the mid cross section; multiply this sum by one sixth the altitude.*

Old Prismoidal Formula. —

$$V = \tfrac{1}{6}\, a(B_1 + 4\,M + B_2).$$

(2) To find the volume of a prismatoid, or of any solid whose section gives a quadratic.

Rule. — *Multiply one fourth its altitude by the sum of one base and three times a section distant from that base two thirds the altitude.*

New Prismoidal Formula. —

$$V = \frac{a}{4}(B + 3\,T).$$

— From Halsted's "Metrical Geometry."

PYRAMID

A pyramid is a polyedron of which all the faces except one meet in a point.

(1) To find the lateral area of a regular pyramid.

Rule. — *Multiply the perimeter of the base by half the slant height.*

(2) To find the volume of any pyramid.

Rule. — *Multiply the area of the base by one third of the altitude.*

PYRAMID, SPHERICAL

A spherical pyramid is the portion of a sphere bounded by a spherical polygon and the planes of its sides.

Rule. — *Multiply the area of the base by one third of the radius of the sphere.*

NOTE. — The area of a spherical polygon is equivalent to a lune whose angle is half the spherical excess of the polygon.

QUADRILATERAL

A quadrilateral is a polygon of four sides.

To find the area of any quadrilateral.

Rule. — *Multiply half the diagonal by the sum of the perpendiculars upon it from the opposite angle.*

RHOMBUS

A rhombus is a parallelogram whose sides are all equal and whose angles are oblique.

To find the area of a rhombus.

Rule. — *Take half the product of its diagonals.*

RINGS

If a plane curve lying wholly on the same side of a line in its own plane revolves about that line, the solid thus generated is called a ring.

(1) Theorem of Pappus.

(*a*) If a plane curve lying wholly on the same side of a line in its own plane revolves about that line, the area of the solid thus generated is equal to the product of the length of the revolving line and the path described by its center of mass.

(*b*) If a plane figure lying wholly on the same side of a line in its own plane revolves about that line, the volume of the solid thus generated is equal to the product of the revolving area and the length of the path described by its center of mass.

(2) To find the surface of an elliptic ring.

Formula. — Surface $= 2\pi^2 c\sqrt{\frac{1}{2}((2a)^2+(2b)^2)}$.

(3) To find the volume of an elliptic ring.

Formula. — Volume $= 2\pi^2 abc$, where $2a$ and $2b$ are the axes of the ellipse and c the distance of the center of the ellipse from the axis of rotation.

(4) To find the surface of a cylindric ring.

Formula. — Surface $= 4\pi^2 ra$.

(5) To find the volume of a cylindric ring.

Formula. — Volume $= 2\pi^2 r^2 a$, where $a =$ distance of the center of the generating curve from the axis of rotation, and $r =$ the radius of the circle.

Roofing and Flooring

A square 10 feet on a side, or 100 square feet, is the unit of measure in roofing, tiling, and flooring.

The average shingle is taken to be 16 inches long and 4 inches wide. Shingles are usually laid about 4 inches to the weather.

When laid $4\frac{1}{2}$ inches to the weather, the exposed surface of a shingle is 18 square inches.

Allowing for waste, about 1000 shingles are estimated as needed for each square, but if the shingles are good, 850 to 900 are sufficient. There are 250 shingles in a bundle.

Sector

A sector is that portion of a circle bounded by two radii and the intercepted arc.

To find the area of a sector.

Rule. — (*a*) *Multiply the length of the arc by half the radius.*

(*b*) *If the arc is given in degrees, take such a part of the whole area of the circle as the number of degrees in the arc is of 360°.*

SECTOR, A SPHERICAL

A spherical sector is the volume generated by any sector of a semicircle which is revolved about its diameter.

To find the volume of a spherical sector.

Rule. — *Multiply the area of its zone by one third the radius.*

Formula. — $V = \frac{2}{3}\pi ar^2$, where r = radius of the sphere and a = altitude of the spherical segment.

SEGMENT OF CIRCLE

A segment of a circle is the portion of a circle included between an arc and its chord.

(1) To find the area of a segment less than a semicircle.

Rule. — *From the sector having the same arc as the segment subtract the area of the triangle formed by the chord and the two radii from its extremities.*

(2) An approximate rule for finding the area of a segment.

Rule. — *Take two thirds the product of its chord and height.*

(3) To find the area of a segment of a circle, having given the chord of the arc and the height of the segment, *i.e.* the versed sine of half the arc.

Rule. — *Divide the cube of the height by twice the base and increase the quotient by two thirds of the product of the height and base.*

(4) To find the volume of the solid generated by a circular segment revolving about a diameter exterior to it.

Rule. — *Multiply one sixth the area of the circle whose radius is the chord of the segment by the projection of that chord upon the axis.*

Formula. — $V = \frac{1}{6}\pi \overline{AB}^2 \times A'B'$, where AB is the chord of the segment and $A'B'$ is its projection upon the axis.

SEGMENT, A SPHERICAL

A spherical segment is a portion of a sphere contained between two parallel planes.

To find the volume of any spherical segment.

Rule. — *To the product of one half the sum of its bases by its altitude add the volume of a sphere having that altitude for its diameter.*

SHELL, A CYLINDRIC

A cylindric shell is the difference between two circular cylinders of the same length.

To find the volume of a cylindric shell.

Rule. — *Multiply the sum of the inner and outer radii by their difference, and this product by* π *times the altitude of the shell.*

SHELL, A SPHERICAL

A spherical shell is the difference between two spheres which have the same center.

To find the volume of a spherical shell.

Formula. — $V = \frac{4}{3}\pi(r_1^3 - r^3)$, where r_1 and r denote the radii.

SIMILAR SOLIDS

Similar solids are solids which have the same form, and differ from each other only in volume.

Rule. — *Any two similar solids are to each other as the cubes of any two like dimensions.*

SIMILAR SURFACES

Similar surfaces are surfaces which have the same shape, and differ from each other only in size.

Rule. — *Any two similar surfaces are to each other as the squares of any two like dimensions.*

SPHERE

A sphere is a closed surface all points of which are equally distant from a fixed point within called the center.

(1) Formulæ. —

$$\text{Area} = 4\,\pi r^2, \text{ or } \pi d^2.$$
$$\text{Area} = r^2 \times 12.5664.$$
$$\text{Area} = d^2 \times 3.1416.$$
$$\text{Area} = \text{circumference}^2 \times .3183.$$
$$\text{Volume} = \tfrac{4}{3}\,\pi r^3, \text{ or } \tfrac{1}{6}\,\pi d^3.$$
$$\text{Volume} = \tfrac{1}{6}\,d \times \text{area}.$$
$$\text{Volume} = \text{circumference}^3 \times .0169.$$
$$\text{Volume} = r^3 \times 4.1888, \text{ or } d^3 \times .5236.$$

(2) Side of an inscribed cube $= \begin{cases} r \times 1.1547, \\ \quad\text{or} \\ d \times .5774. \end{cases}$

(3) To find the edge of the largest cube that can be cut from a hemisphere.

Formula. — Edge $= d \times .408248$.

(4) To find the volume of a frustum of a sphere, or the portion included between two parallel planes.

Rule. — *To three times the sum of the squared radii of the two ends add the square of the altitude; multiply this sum by .5235987 times the altitude.*

(5) To find the edge of the largest cube that can be inscribed in a hemisphere of given diameter.

Rule. — *Multiply the radius by $\frac{1}{3}$ of the square root of 6.*

SPHEROID

A spheroid is a solid formed by revolving an ellipse about one of its axes as an axis of revolution.

SPHEROID, OBLATE

An oblate spheroid is the spheroid formed by revolving an ellipse about its conjugate diameter as an axis of revolution.

To find the volume of an oblate spheroid.

Rule. — *Multiply the square of the semitransverse diameter by the semiconjugate diameter and this product by* $\frac{4}{3}\pi$.

SPHEROID, PROLATE

A prolate spheroid is the spheroid formed by revolving an ellipse about its transverse diameter as an axis of revolution.

To find the volume of a prolate spheroid.

Rule. — *Multiply the square of the semiconjugate diameter by the semitransverse diameter and this product by* $\frac{4}{3}\pi$.

SPINDLE, A CIRCULAR

A circular spindle is the solid formed by revolving the segment of a circle about its chord.

(1) To find the volume of a circular spindle.

Rule. — *Multiply the area of the generating segment by the path of its center of gravity.*

(2) To find the volume formed by revolving a semicircle about a tangent parallel to its diameter.

Rule. — *Multiply one fourth of the volume of a sphere whose radius is that of the generating semicircle by* $(10 - 3\pi)$.

SPINDLE, A PARABOLIC

A parabolic spindle is a solid formed by revolving a parabola about a double ordinate perpendicular to the axis.

To find the volume of a parabolic spindle.

Rule. — *Multiply the volume of its circumscribed cylinder by* $\frac{8}{15}$.

SQUARE

A square is a rectangle whose sides are all equal.

(1) To find the area of a square.

Rule. — *Square an edge.*

(2) Given the diagonal, to find the area.

Rule. — *Take one half the square of the diagonal.*

(3) Given the diagonal, to find a side.

Rule. — *Extract the square root of one half the square of the diagonal.*

(4) To find the side of the largest square inscribed in a semicircle of given diameter.

Rule. — *Multiply the radius of the given circle by $\frac{2}{5}$ of the square root of 5.*

TETRAEDRON

A tetraedron is a polyedron of four faces.

(1) To find the surface of a tetraedron.

Rule. — *Multiply the square of an edge by $\sqrt{3}$, or 1.73205.*

(2) To find the volume of a tetraedron.

Rule. — *Multiply the cube of an edge by $\frac{1}{12}\sqrt{2}$,* or *.11785.*

TRAPEZIUM AND IRREGULAR POLYGONS

To find the area of a trapezium or any irregular polygon.

Rule. — *Divide the figure into triangles, find the area of the triangles, and take their sum.*

TRAPEZOID

A trapezoid is a quadrilateral two of whose sides are parallel.

(1) To find the area of a trapezoid.

Rule. — *Multiply the altitude by one half the sum of the parallel sides.*

(2) Width = area ÷ ($\frac{1}{2}$ of the sum of the parallel sides).

(3) Sum of the parallel sides = (area ÷ width) × 2.

(4) To find the length of a line parallel to the bases of a trapezoid that shall divide it into equal areas.

Rule. — *Square the bases and extract the square root of half their sum.*

TRIANGLE

A triangle is a portion of a plane bounded by three straight lines.

(1) To find the area of a triangle.

Rule. — *Multiply the base by half the altitude.*

(2) To find the area of a triangle, having given the three sides.

Rule. — *From half the sum of the three sides subtract each side separately; multiply half the sum and the three remainders together: the square root of the product will be the area.*

(3) To find the radius of the inscribed circle.

Rule. — *Divide the area of the triangle by half the sum of its sides.*

(4) To find the radius of the circumscribing circle.

Rule. — *Divide the product of the three sides by four times the area of the triangle.*

(5) To find the radius of an escribed circle.

Rule. — *Divide the area of the triangle by the difference between half the sum of its sides and the tangent side.*

(6) To cut off a triangle containing a given area by a line running parallel to one of its sides, having given the area and base.

Rule. — *The area of the given triangle is to the area of the triangle to be cut off, as the square of the given base is to the square of the required base. Extract the square root of the result.*

(*Equilateral*) *Triangle*

(1) Area = one half the side squared and multiplied by $\sqrt{3}$, or 1.732050+.

(2) Altitude = one half the side multiplied by $\sqrt{3}$, or 1.732050^{+}.

(3) Center of the inscribed and circumscribed circle is a point in the altitude one third of its length from the base.

(4) Radius of the circumscribed circle = two thirds of the altitude.

(5) Radius of the inscribed circle = one third of the altitude.

(6) Side $= 2\sqrt{\text{area} \div \sqrt{3}}$.

Side = radius of the circumscribed circle multiplied by $\sqrt{3}$.

(7) All equilateral triangles are similar.

(8) Each angle = 60°.

(*Right*) *Triangle*

(1) Base $= \sqrt{(h^2 - p^2)}$.

(2) Perpendicular $= \sqrt{(h^2 - b^2)}$.

(3) Hypotenuse $= \sqrt{b^2 + p^2}$.

(4) Diameter of inscribed circle $= (b + p) - h$.

(5) Side opposite an acute angle of 30° = one half of the hypotenuse.

(6) Similar, if an acute angle of one = an acute angle of another.

(7) Altitude of an isosceles triangle forms two right triangles.

(8) To find a point in a right-angled triangle equidistant from its vertices.

Rule. — *Divide the hypotenuse by 2; the point will lie in the hypotenuse.*

(9) To find the perpendicular height of a right triangle when the base and the sum of the perpendicular and hypotenuse are known.

Rule. — *From the square of the sum of the perpendicular and hypotenuse take the square of the base, and divide the difference by twice the sum of the perpendicular and hypotenuse.*

(SPHERICAL) TRIANGLE

A spherical triangle is a spherical polygon of three sides.

To find the area of a spherical triangle.

Rule. — *Find the area of a lune whose angle is half the spherical excess of the triangle.*

NOTE. — The spherical excess of a triangle is the excess of the sum of its angles over 180°.

UNGULA, A CONICAL

A conical ungula is a portion of a cone cut off by a plane oblique to the base and contained between this plane and the base.

To find the volume of a conical ungula, when the cutting plane passes through the opposite extremes of the ends of the frustum.

Rule. — *Multiply the difference of the square roots of the cubes of the radii of the bases by the square root of the cube of the radius of the lower base and this product by $\frac{1}{3}\pi$ times the altitude. Divide this last product by the difference of the radii of the two bases, and the quotient will be the volume of the ungula.*

UNGULA, A CYLINDRIC

A cylindric ungula is any portion of a cylinder cut off by a plane.

(1) To find the convex surface of a cylindric ungula, when the cutting plane is parallel to the axis of the cylinder.

Rule. — *Multiply the arc of the base by the altitude.*

(2) To find the volume of a cylindric ungula whose cutting plane is parallel to the axis.

Rule. — *Multiply the area of the base by the altitude.*

(3) To find the convex surface of a cylindric ungula, when the plane passes obliquely through the opposite sides of the cylinder.

Rule. — *Multiply the circumference of the base by half the sum of the greatest and least lengths of the ungula.*

(4) To find the volume of a cylindric ungula, when the plane passes obliquely through the opposite sides of the cylinder.

Rule. — *Multiply the area of the base by half the least and greatest lengths of the ungula.*

UNGULA, A SPHERICAL

A spherical ungula is a portion of a sphere bounded by a lune and two great semicircles.

To find the volume of a spherical ungula.

Rule. — *Multiply the area of the lune by one third the radius; or, multiply the volume of the sphere by the quotient of the angle of the lune divided by 360°.*

WEDGE

A wedge is a prismatoid whose lower base is a rectangle, and upper base a sect parallel to a basal edge.

To find the volume of any wedge.

Rule. — *To twice the length of the base add the opposite edge; multiply the sum by the width of the base, and this product by one sixth the altitude of the wedge.*

WOOD MEASURE

The unit of wood measure is the cord. The cord is a pile of wood 8 feet by 4 feet by 4 feet.

A pile of wood 1 foot by 4 feet by 4 feet is called a cord foot.

A cord of stove wood is 8 feet long by 4 feet high. The length of stove wood is usually 16 in.

ZONE

A zone is the curved surface of a sphere included between two parallel planes or cut off by one plane.

(1) To find the area of a zone.

Rule. — *Multiply the altitude of the spherical segment by twice* π *times the radius of the sphere.*

(2) To find the area of a zone of one base.

Rule. — *The area of a zone of one base is equivalent to the area of a circle whose radius is the chord of the generating arc.*

(CIRCULAR) ZONE

A circular zone is the portion of a plane inclosed by two parallel chords and their intercepted arcs.

(1) If both chords are on the same side of the center.

Rule. — *Find the difference between the areas of the two segments.*

(2) If the chords are on opposite sides of the center.

Rule. — *Subtract the sum of the areas of the two segments from the area of the circle.*

MISCELLANEOUS HELPS

1. Pi (π) = 3.1416, or $3\frac{1}{7}$. Its value to seven hundred and seven places is

3.14159265358979323846264338327950288419716939937510582
097494459230781640628620899862803482534211706798214808
651328230664709384460955058223172535940812848111745028
410270193852110555964462294895493038196442881097566593
344612847564823378678316527120190914564856692346034861
045432664821339360726024914127372458700660631558817488
152092096282925409171536436789259036001133053054882046
652138414695194151160943305727036575959195309218611738
193261179310511854807446237983474956735188575272489122
793818301194912983367336244193664308602139501609244807
723094362855309662027556939798695022247499620607497030
412366886199511008920238377021314169411902988582544681
639799904659700081700296312377381342084130791451183980
570985.

2. The contents of a spheroid equals the square of the revolving axis × the fixed axis × .5236.

3. To find the distance a spot on the tire of a revolving wheel moves, multiply the distance traveled by 4 and divide by π.

4. Sound travels 1087 feet per second at 0° C. or 1126 feet per second at 20° C.

5. Electricity travels about 186,000 miles per second.

6. To find the approximate number of bushels of corn in a crib, take the dimensions in feet, and multiply their product

by .8, if the corn is shelled; by .4, if shucked; by .3, if in the shuck.

7. Roofing, flooring, and slating are often estimated by the square, which contains 100 square feet.

8. The long ton of 2240 pounds and the long hundredweight of 112 pounds are used in the United States custom houses and in weighing coal and iron in the mines.

9. The term carat is sometimes used to express the fineness of gold, each carat meaning a twenty-fourth part.

10. It takes 1000 shingles to cover 100 square feet laid 4 inches to the weather. It takes 900 shingles to cover 100 square feet laid $4\frac{1}{2}$ inches to the weather.

11. The area of an ellipse is a mean proportional between the circumscribed and inscribed circles.

12. Gunter's chain is 66 feet long, consisting of 100 links.

13. The first 24 periods of numeration are — units, thousands, millions, billions, trillions, quadrillions, quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, tredecillions, quartodecillions, quintodecillions, sexdecillions, septodecillions, octodecillions, nonodecillions, vigintillions, primo-vigintillions, and secundo vigintillions.

14. Mathematicians have given the signs × and ÷ precedence over the signs + and − ; hence the operations of multiplication and division should always be performed before addition and subtraction.

15. The true weight of an article weighed on false scales is a mean proportional between the two apparent weights.

16. To find any term of an arithmetical progression.

Rule. — *Any term of an arithmetical series is equal to the first term, increased or diminished by the common difference multiplied by a number one less than the number of terms.*

17. To find the sum of an arithmetical series.

Rule. — *Multiply half the sum of the extremes by the number of terms.*

18. To find any term of a geometrical series.

Rule. — *Multiply the first term by the ratio raised to a power one less than the number of terms.*

19. To find the sum of a geometrical series.

Rule. — *Multiply the greater extreme by the ratio, subtract the less extreme from the product, and divide the remainder by the ratio less 1.*

20. To sum a geometrical series to infinity.

Rule. — *When the ratio is a proper fraction, divide the first term by 1 less the ratio.*

21. To find the harmonic mean between two numbers.

Rule. — *Divide twice their product by their sum.*

22. To find the mean proportional between two numbers.

Rule. — *Take the square root of their product.*

23. A body immersed in a liquid is buoyed up by a force equal to the weight of the liquid displaced. That is, it loses a portion of its weight just equal to the weight of the water displaced.

24. If we have the sum and difference of two numbers given, add the sum and difference and take half of it for the greater, subtract and take half of it for the smaller.

25. To find the day of the week for any date.

Rule. — *To the given year of the century add its $\frac{1}{4}$, neglecting remainder; to this add the day of the month, the ratio of the century, and the ratio of the month; then divide by 7, and the remainder will be the number of the day of the week, counting Sunday 1st, Monday 2d, and so on.*

CENTENNIAL RATIO	MONTHLY RATIOS	
200, 900, 1800, 2200 = 0.	January = 3 or 2.	August = 5.
300, 1000 . . . . . = 6.	February = 6 or 5.	September = 1.
400, 1100, 1900, 2300 = 5.	March = 6.	October = 3.
500, 1200, 1600, 2000 = 4.	April = 2.	November = 6.
600, 1300 . . . . . = 3.	May = 4.	December = 1.
700, 1400, 1700, 2100 = 2.	June = 0.	In leap years
100, 800, 1500 . . = 1.	July = 2.	Jan. = 2. Feb. = 5.

Examples. — March 4, 1877, was on $[77+19+4+0+6] \div 7$, remainder $1 =$ *Sunday*. Jan. 31, 1845 was on $[45+11+31+0+3] \div 7$, remainder $6 =$ *Friday*. Oct. 12, 1492, was on $[92+23+12+2+3] \div 7$, remainder $6 =$ *Friday*. Leap years are known by being divisible by 4, except those centuries that cannot be divided by 400; hence 1900 was not a leap year.

26. To find the day's length at any latitude (for example, 71° N. Lat.).

Let t be the time before 6 o'clock for sunrise; then the length of the day is $(2t$ plus $12)$ hours. If d be the sun's declination and l the latitude, then $\sin \frac{1}{4}t$ equals $\cot (90° - l) \tan d$. For longest day d equals $23° 27'$, and l equals $71°$. Therefore, $\sin \frac{1}{4}t$ equals $\cot 19° \tan (23° 27')$. $\frac{1}{4}t$ must be expressed in degrees.

$$\begin{array}{r} \log \cot 19° = 10.463028 \\ \log \tan (23° 27') = 9.637265 \\ \hline \log \frac{1}{4}t = 10.100293 \end{array}$$

As the logarithm of the sine of an angle cannot be greater than 10, this shows that the person's latitude is within the limits of the Arctic circle, and on the longest day there the sun does not rise and set. — From "The School Visitor."

27. To find the G. C. D. of fractions.

Rule. — *Find the G. C. D. of the numerators of the fractions, and divide it by the L. C. M. of their denominators.*

28. To find the L. C. M. of fractions.

Rule. — *Divide the L. C. M. of the numerators by the G. C. D. of the denominators.*

29. To find the height of a stump of a broken tree.

Rule. — *From the square of the height of the tree subtract the square of the distance the top rests from the base of the tree, and divide the remainder by twice the height of the tree.*

30. To find how many board feet in a round log.

Rule. — *Subtract 4 from the diameter of the log in inches, and the square of this remainder equals the number of board feet in a log 16 feet long.*

31. To find the velocity of a nailhead in the rim of a moving wheel.

Rule. — *Divide twice the height of the nailhead above the plane upon which the wheel rolls, by the radius, and multiply this product by the velocity of the center; then extract the square root.*

Note. — Its velocity at the bottom is zero; at the top, twice that of the center; and when its height is half the radius, its velocity equals that of the center.

32. To find the distance to the horizon.

Rule. — *Take one and one half times the height the observer is above the surface of the ground in feet. The square root of this number is the number of miles an object on the surface can be seen.*

33. Extraction of any root.

Horner's Method, invented by Mr. Horner, of England, is the best general method of extracting roots.

Any root whose index contains only the factors 2 or 3 can be extracted by means of the square and cube root.

Rule. — I. *Divide the number into periods of as many figures each as there are units in the index of the root, and at the left of the given number arrange the same number of columns, writing 1 at the head of the left-hand column and ciphers at the head of the others.*

II. *Find the required root of the first period, for the first figure of the root, multiply the number in the 1st col. by this first term of the root and add it to the 2d col., multiply this sum by the root and add it to the 3d col., and thus continue, writing the last prod-*

uct under the first period; subtract and bring down the next period for a dividend.

III. *Repeat this process, stopping one column sooner at the right each time until the sum falls in the 2d col. Then divide the dividend by the number in the last column, which is the trial divisor; the result is the second figure of the root.*

IV. *Use the second figure of the root precisely as the first, remembering to place the products one place to the right in the 2d col., two in the 3d col., etc.; continue this operation until the root is completed or carried as far as desired.*

NOTES. — 1. Only a part of the dividend is used for finding a root figure, according to the principle of place value. The partial dividend thus used always terminates with the first figure of the period annexed.

2. If any dividend does not contain the trial divisor, place a cipher in the root, and bring down the next period; annex one cipher to the last term of the 2d column, two ciphers at the last term of the 3d, three to the 4th, and then proceed according to the rule.

Example. — Extract the fourth root of 5636405776.

OPERATION

1	0	0	0	56·3640·5776(274
	2	4	8	16
	2	8	24	403640
	4	12	(1) 32 t. d.	
	2	12	21063	
	6	(1) 24	53063 T. D.	371441
	2	609	25669	321995776
	(1) 8	3009	(2) 78732 t. d.	
	7	658	1766944	
	87	3667	80498944 T. D.	321995776
	7	707		
	94	(2) 4374		
	7	4336		
	101	441736		
	7			
	(2) 108			
	4			
	1084			

— From Brooks' "Higher Arithmetic."

SCIENTIFIC TRUTHS

1. The intensity of light varies inversely as the square of the distance from the source of illumination.

2. The intensity of sound varies inversely as the square of the distance from the source of the sound.

3. Gravitation varies inversely as the square of the distance between the centers of gravity.

4. The heating effect of a small radiant mass upon a distant object varies inversely as the square of the distance.

MATHEMATICAL DEFINITIONS

Algebra is that branch of mathematics in which mathematical investigations and computations are made by means of letters and other symbols.

Analytical Geometry is that branch of geometry in which the properties and relations of geometrical magnitudes are investigated by the aid of algebraic analysis.

Analytical Trigonometry is that branch of trigonometry which treats of the properties and relations of the trigonometrical functions.

Applied, or **Mixed, Mathematics** is the application of pure mathematics to the mechanic arts.

Arithmetic is the science that treats of numbers, the methods of computing by them, and their applications to business and science.

Astronomy is that branch of applied mathematics in which mathematical principles are used to explain astronomical facts.

Calculus is that branch of algebraic analysis which commands, by one general method, the most difficult problems of geometry and physics.

Calculus of Variations is that branch of calculus in which the laws of dependence which bind the variable quantities together are themselves subject to change.

Conic Sections is that branch of Platonic geometry which treats of the curved lines formed by the intersection of the surface of a right cone and a plane.

Descriptive Geometry is that branch of geometry which treats of the graphic solutions of all problems involving three dimensions by means of projections upon auxiliary planes.

Differential Calculus is that branch of calculus which investigates mathematical questions by using the ratio of certain indefinitely small quantities called differentials.

Geometry is the science which treats of the properties and relations of space.

Gunnery is that branch of applied mathematics which treats of the theory of projectiles.

Integral Calculus is that branch of calculus which determines the relations of magnitudes from the known differentials of these magnitudes. It is the reverse method of the differential calculus.

Mathematics is that science which treats of the measurement of and exact relations existing between quantities and of the methods by which it draws necessary conclusions from given premises.

Mechanics is that branch of applied mathematics which treats of the action of forces on material bodies.

Mensuration is that branch of applied mathematics which treats of the measurement of geometrical magnitudes.

Metrical Geometry is that branch of geometry which treats of the length of lines and the magnitudes of angles, areas, and solids.

Navigation is that branch of applied mathematics which treats of the art of conducting ships or vessels from one place to another.

Optics is that branch of applied mathematics which treats of the laws of light.

Plane Geometry is that branch of pure geometry which treats of figures that lie in the same plane.

Plane Trigonometry is that branch of trigonometry which treats of the solution of plane triangles.

Platonic Geometry is that branch of metrical geometry in which the argument, or proof, is carried forward by a direct inspection of the figures themselves, or pictured before the eye in drawings, or held in the imagination.

Pure Geometry is that branch of Platonic geometry in which the argument, or proof, uses compasses and ruler only.

Pure Mathematics treats of the properties and relations of quantity without relation to material bodies.

Quaternions is that branch of algebra which treats of the relations of magnitude and position of lines or bodies in space by means of the quotient of two vectors, or of two directed right lines in space, considered as depending on four geometrical elements, and as expressible by an algebraic symbol of quadrinomial form.

Solid Geometry, or **Geometry of Space**, is that branch of pure geometry which treats of figures which do not lie wholly within the same plane.

Spherical Trigonometry is that branch of trigonometry which treats of the solution of spherical triangles.

Surveying is that branch of applied mathematics which teaches the art of determining and representing areas, lengths and directions of bounding lines, and the relative position of points upon the earth's surface.

Trigonometry is that branch of Platonic geometry which treats of the relations of the angles and sides of triangles.

HISTORICAL NOTES

The oldest known mathematical work, a papyrus manuscript deciphered in 1877, and preserved in the British Museum, was written by Ash-mesu (the moon-born), commonly called Ahmes, an Egyptian, sometime before 1700 B.C. This work was entitled "Directions for obtaining the Knowledge of All Dark Things." This work contains problems in arithmetic and geometry and contains the first suggestions of algebraic notation and the solution of equations. This work was founded on another work believed to date back as far as 3400 B.C.

Pythagoras, who died about 580 B.C., raised mathematics to the rank of a science. He was one of the most remarkable men of antiquity.

The study of geometry was introduced into Greece about 600 B.C. by Thales. Thales founded a school of mathematics and philosophy at Miletus, known as the Ionic School.

Euclid's "Elements," the greatest textbook on geometry, was published about 300 B.C. Euclid taught mathematics in the great university at Alexandria, Egypt.

The name Mathematics is said to have first been used by the Pythagoreans.

About 440 B.C. Hippocrates of Chios wrote the first Greek textbook on geometry.

To the great philosophic school of Plato, which flourished at Athens (429–348 B.C.), is due the first systematic attempt to create exact definitions, axioms, and postulates, and to distinguish between elementary and higher geometry.

Diophantus, who died about 330 A.D., was the first writer on algebra worthy of recognition. His "Arithmetica" is the earliest treatise on algebra now extant. He was the first to state that "a negative number multiplied by a negative number gives a positive number."

Al Hovarezmi, who died about 830, published the first book known to contain the word "algebra" in the title.

The first edition of Euclid was printed in Latin in 1482, and the first one in English appeared in 1570.

Robert Recorde published the first arithmetic printed in the English language in 1540.

The first arithmetic published in America was written by Isaac Greenwood and issued in 1729.

Chauncey Lee published in 1797 an arithmetic called "The American Accomptant." This work contains the dollar mark, though in much ruder form than the character now in use.

Descartes, the French philosopher, invented the method of computing graphs from equations about 1637. On June 8, 1637, he published the first analytical geometry.

The differential calculus was invented by Newton and Leibniz about 1670.

In 1686 Leibniz published in a paper, "The Acta Eruditorum," the rudiments of the integral calculus.

Hipparchus, who lived sometime between 200 and 100 B.C., was the greatest astronomer of antiquity and originated the science of trigonometry.

The symbols of the Hindu or Arabic notation, except the zero, originated in India before the beginning of the Christian era. The zero appeared about 500 A.D.

Nearly 4000 years ago Ahmes solved problems involving the area of the circle and found results that gave $\pi = 3.1604$. The Babylonians and Jews used $\pi = 3$. The Romans used 3 and sometimes 4, or for more accurate work $3\frac{1}{8}$. About 500 A.D. the Hindus used 3.1416. The Arabs about 830 A.D. used $\frac{22}{7}$, $\sqrt{10}$, 3.1416. In 1596 Van Ceulen computed π to over 30 decimal places. In 1873 Shanks computed π to 707 decimal places.

Logarithms were invented by John Napier, of Scotland, about 1614 A.D. His logarithms were not of ordinary numbers, but of the ratios of the legs of a right-angled triangle to the hypotenuse.

Later Briggs constructed tables of logarithmic numbers to the base 10.

The first publication of Briggian logarithms of trigonometric functions was made in 1620 by Gunter. Gunter was a colleague of Briggs. He invented the words cosine and cotangent, and found the logarithmic sines and tangents for every minute to seven places.

HISTORICAL NOTES ON ARITHMETIC

"The Science of Arithmetic is one of the purest products of human thought. Based upon an idea among the earliest which spring up in the human mind, and so intimately associated with its commonest experience, it became interwoven with man's simplest thought and speech, and was gradually unfolded with the development of the race. The exactness of its ideas, and the simplicity and beauty of its relations, attracted the attention of reflective minds, and made it a familiar topic of thought; and, receiving contributions from age to age, it continued to develop until it at last attained to the dignity of a science, eminent for the refinement of its principles and the certitude of its deductions.

"The science was aided in its growth by the rarest minds of antiquity, and enriched by the thought of the profoundest thinkers. Over it Pythagoras mused with the deepest enthusiasm; to it Plato gave the aid of his refined speculations; and in unfolding some of its mystic truths, Aristotle employed his peerless genius. In its processes and principles shines the thought of ancient and modern mind — the subtle mind of the Hindu, the classic mind of the Greek, the practical spirit of the Italian and English. It comes down to us adorned with the offerings of a thousand intellects, and sparkling with the gems of thought received from the profoundest minds of nearly every age." — From Brooks' "Philosophy of Arithmetic."

The first step in the historical development of arithmetic was in counting things. How far back this operation dates is not known. Counting among primitive people was of a very elementary nature, as it is now among people of a low grade of civilization. A knowledge of arithmetic is coeval with the race. Every people, no matter how uncivilized, has some crude knowledge of numbers and employs them in its transactions with one another. Some of them have no real numeral words, while others have very few. The Chiquitos of Bolivia have no real numerals. The Campas of Peru have only three, but can count to ten. The Bushmen of South Africa have but two numerals. The natives of Lower California cannot count above five. Very few of the Esquimos can count above five. The more intelligent can count to twenty or more.

The Egyptians stand at the beginning of the first period in the historical development of arithmetic. Menes, their first king, changed the course of the Nile, made a great reservoir, and built the temple of Phthah at Memphis. They built the pyramids at a very early period. Surely a people who were engaged in enterprises of such magnitude must have known something of mathematics — at least of practical arithmetic. To them all Greek writers are unanimous in ascribing, without envy, the priority of invention in the mathematical sciences.

Aristotle says that mathematics had its birth in Egypt, because there the priestly class had the leisure needful for the study of it. In Herodotus we find this (11 c 109): "They said also that this king (Sesostris) divided the land among all Egyptians so as to give each one a quadrangle of equal size and to draw from each his revenues, by imposing a tax to be levied yearly. But every one from whose part the river tore away anything, had to go to him and notify what had happened; he then sent the overseers, who had to measure out by how much the land had become smaller, in order that the owner might pay on what was left, in proportion to the entire tax imposed. In this way, it appears to me, geometry originated."

One of the oldest known works on mathematics, a manuscript copied on papyrus, a kind of paper used about the Mediterranean in early times, is still preserved and is now in the British Museum. It was deciphered in 1877 and found to be a mathematical manual containing problems in arithmetic and geometry. It was written by Ahmes sometime before 1700 B.C., and was founded on an older work believed to date back as far as 3400 B.C. This work is entitled "Directions for obtaining the Knowledge of All Dark Things." In the arithmetical part it teaches operations with whole numbers and fractions. Some problems in this papyrus seem to imply a rudimentary knowledge of proportion. The area of an isosceles triangle, of which the sides measure 10 ruths and the base 4 ruths, is erroneously given as 20 square ruths, or half the product of the base by one side. The area of a circle is found by deducting from the diameter $\frac{1}{9}$ of its length and squaring the remainder. π is taken $= (\frac{16}{9})^2 = 3.1604$.

According to Herodotus the ancient Egyptian computation consisted in operating with pebbles on a reckoning board whose lines were at right angles to the user. There is reason to believe the Babylonians used a similar device. The earliest Greeks, like the Egyptians and Eastern nations, counted on the fingers or with pebbles. The Romans employed three methods, reckoning upon the fingers, upon the abacus (a mechanical contrivance with columns for counters), and by tables prepared for the purpose. The method of finger reckoning seems to have prevailed among savage tribes from the beginning of time, and every observer knows how exceedingly common its use is among children learning to count. They perhaps adopt this method instinctively.

The Egyptians used the decimal scale. The Greeks and Egyptians made exclusive use of unit fractions, or fractions having one for the numerator. They kept the numerator constant and dealt with variable denominators. The Babylonians kept the denominators constant and equal to 60. Also the Romans kept them constant, but equal to 12.

The Greeks also had much to do with the advancement of mathematics. They discriminated between the science of numbers and the art of calculation. They were among the first writers on arithmetic. About twenty-five centuries ago Pythagoras classified numbers into perfect and imperfect, even and odd, solid, square, cubical, etc. "He regarded numbers as of divine origin — the fountain of existence — the model and archetype of things — the essence of the universe." He regarded even numbers as feminine, and allied to the earth; odd numbers were supposed to be endued with masculine virtues, and partook of the celestial nature. He considered "number as the ruler of forms and ideas, and the cause of gods and dæmons"; and again that "to the most ancient and all-powerful creating Deity, number was the canon, the efficient reason, the intellect also, and the most undeviating of the composition and generation of all things."

Philolaus declared "that number was the governing and self-begotten bond of the eternal permanency of mundane natures." Another ancient said that number was the judicial instrument of the Maker of the universe, and the first paradigm of mundane fabrication.

Plato ascribed the invention of numbers to God himself. In the "Phædrus" he said, "The name of the Deity himself was Theuth. He was the first to invent numbers, and arithmetic, and geometry, and astronomy." In the "Timæus," he said, "Hence, God ventured to form a certain movable image of eternity; and thus while he was disposing the parts of the universe, he, out of that eternity which rests in unity, formed an eternal image on the principle of numbers, and to this we give the appellation of time."

Euclid, who lived about 300 B.C., was one of the early Greek writers upon arithmetic. In his "Elements" he treats of the theory of numbers, including prime and composite numbers, greatest common divisor, least common multiple, continued proportion, geometrical progressions, etc.

Archimedes, who was born about 287 B.C., was one of the most noted Greek mathematicians. He discovered the ratio of the cylinder to the inscribed sphere, and in commemoration of this the figure of a cylinder was engraved upon his tomb. He also wrote two papers on arithmetic. In the first he explained a convenient system of representing large numbers. In the second he showed that this method enabled a person to write any number however large, and as proof gave his celebrated illustration that the number of grains of sand required to fill the universe is less than 10^{63}.

In 1202 Leonardo of Pisa published his great work "Liber Abaci." This work contained about all the knowledge the Arabs possessed in arithmetic and algebra and furnished the most lasting material for the extension of Hindu methods.

In 1540 Robert Recorde published the first arithmetic printed in the English language. He invented the present method of extracting the square root.

In 1729 Isaac Greenwood published the first arithmetic published in America.

In 1788 Nicolas Pike's arithmetic was published at Newburyport, Mass. It was a very popular book and was highly recommended by George Washington.

In 1797 Chauncey Lee published "The American Accomptant."

In 1799 Daboll published at New London, Conn., "The Schoolmaster's Assistant," which was indorsed by Noah Webster. In this book the comma is used in place of the decimal point.

In 1821 Warren Colburn's "First Lessons in Intellectual Arithmetic" appeared. This book met with remarkable success. About two million copies were sold in twenty-five years. It revolutionized the teaching of arithmetic, and its influence is felt to this day.

MATHEMATICAL SIGNS

The symbols $+$ and $-$ were used by Widmann in his arithmetic published at Leipzig in 1489, $=$ by Robert Recorde in his "Whetstone of Witte" published in 1557, $\times$ by William Oughtred in 1631, the dot $(\cdot)$ as a symbol of multiplication by Harriot in 1631, the absence of a sign between two letters to indicate multiplication by Stifel in 1544, $:$ as a symbol of division by Leibniz, $/$ as a symbol of division was used very early by the Hindus and Arabs and is supposed to be the oldest of all the mathematical signs, $\div$ as a symbol of division by Rahn, a Swiss, in an algebra published at Zurich in 1659, $>$ and $<$ by Harriot in 1631, $::$ by Oughtred in 1631, $\sqrt{\ }$ was first used in this form by Rudolff in 1525, ∞ and fractional exponents by Wallis and Newton in 1658, dx and $\int$ by Leibniz on October 29, 1675.

The symbols $\neq$, $\not>$, $\not<$, indicating "not equal," etc., are recent. Parentheses were first used as symbols of aggregation by Girard in 1629. The decimal point came into use in the seventeenth century; it seems to have appeared first in a work published by Pitiscus in 1612. Positive integral exponents in the present form were first used by Chuquet in 1484.

The Greek letter π was first used to represent the ratio of the circumference to the diameter by William Jones in his "Synopsis Palmariorum Matheseos," in 1706, and came into general use through the influence of Euler.

THE BRANCHES OF MATHEMATICS CLASSIFIED

- Mathematics
  - 1. Pure
    - 1. Arithmetic
    - 2. Algebra or Analysis
      - 1. Algebra
      - 2. Calculus
        - 1. Calculus of Variations
        - 2. Differential
        - 3. Integral
      - 3. Quaternions
    - 3. Geometry
      - 1. Descriptive Geometry or Projective Geometry
      - 2. Metrical Geometry
        - 1. Platonic Geometry
          - 1. Pure Geometry
            - 1. Plane Geometry
            - 2. Solid Geometry or Geometry of Space
          - 2. Conic Sections
          - 3. Trigonometry
            - 1. Plane Trigonometry
            - 2. Spherical Trigonometry
        - 2. Analytical Geometry
          - 1. Plane Analytical Geometry
          - 2. Solid Analytical Geometry
  - 2. Applied
    - 1. Astronomy
    - 2. Gunnery
    - 3. Mechanics
    - 4. Mensuration
    - 5. Navigation
    - 6. Optics
    - 7. Surveying
    - 8. Etc.

NUMBERS AND FRACTIONS CLASSIFIED

- Numbers
  - 1. Real
    - 1.
      - 1. Integer
        - 1.
          - 1. Abstract
          - 2. Concrete
        - 2.
          - 1. Even
          - 2. Odd
        - 3.
          - 1. Perfect
          - 2. Imperfect
        - 4.
          - 1. Prime
          - 2. Composite
      - 2. Fraction
      - 3. Mixed
    - 2.
      - 1. Rational
      - 2. Irrational
  - 2. Imaginary

- Fractions
  - 1. Common or Vulgar
    - 1. As to form
      - 1. Simple
      - 2. Complex
      - 3. Compound
    - 2. As to value
      - 1. Proper
      - 2. Improper
  - 2. Continued
  - 3. Decimal
    - 1. Pure
    - 2. Circulating
      - 1. Pure
      - 2. Mixed

GEOMETRICAL MAGNITUDES CLASSIFIED

Geometrical Magnitudes
- 1. Lines
  - 1. Straight
    - 1. Parallel
    - 2. Perpendicular
    - 3. Horizontal
    - 4. Vertical
    - 5. Oblique
  - 2. Broken
  - 3. Curved
- 2. Angles
  - 1. Right
  - 2. Oblique
    - 1. Acute
    - 2. Obtuse
  - 3. Straight
  - 4. Reflex
- 3. Surfaces
  - 1. Plane
    - 1. Polygon
      - 1. Triangle
        - 1. Right
          - 1. Isosceles
          - 2. Scalene
        - 2. Oblique
          - 1. Acute
            - 1. Scalene
            - 2. Isosceles
            - 3. Equilateral
          - 2. Obtuse
            - 1. Scalene
            - 2. Isosceles
      - 2. Quadrilateral
        - 1. Parallelogram
          - 1. Rectangle | Square
          - 2. Rhomboid | Rhombus
        - 2. Trapezoid
        - 3. Trapezium
      - 3. Pentagon
      - 4. Hexagon
      - 5. Etc.
    - 2. Circle
    - 3. Ellipse
    - 4. Etc.
  - 2. Curved
    - 1. Spherical
    - 2. Conical
    - 3. Cylindrical
    - 4. Etc.
- 4. Solids
  - 1. Polyedron . . . . .
    - 1. Prism
      - 1. Triangular
      - 2. Quadrangular | Parallelopipedon
        - 1. Right or Rectangular | Cube
        - 2. Oblique
      - 3. Pentagonal
      - 4. Hexagonal
      - 5. Etc.
    - 2. Pyramids and their frusta
      - 1. Triangular
      - 2. Quadrangular
      - 3. Pentagonal
      - 4. Hexagonal
      - 5. Etc.
    - 3. Octaedron
    - 4. Dodecaedron
    - 5. Icosaedron
    - 6. Prismatoid
    - 7. Etc.
  - 2. Cylinder
    - 1. Right
    - 2. Oblique
  - 3. Cones and their frusta
    - 1. Right
    - 2. Oblique
  - 4. Sphere
  - 5. Etc.

MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	156	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

TABLE SHOWING THE AMOUNT OF $1 AT COMPOUND INTEREST FROM 1 YEAR TO 20 YEARS

YR.	2½ PER CENT	3 PER CENT	3½ PER CENT	4 PER CENT	5 PER CENT	6 PER CENT
1	1.025	1.03	1.035	1.04	1.05	1.06
2	1.050625	1.0609	1.071225	1.0816	1.1025	1.1236
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.4071	1.50363
8	1.218403	1.26677	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1 551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.45997	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.51259	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.59865	1.753506	1.922501	2.106849	2.52695	3.0256
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

YR.	7 PER CENT	8 PER CENT	9 PER CENT	10 PER CENT	11 PER CENT	12 PER CENT
1	1.07	1.08	1.09	1.10	1.11	1.12
2	1.1449	1.1664	1.1881	1.21	1.2321	1.2544
3	1.225043	1.259712	1.295029	1.331	1.367631	1.404908
4	1.310796	1.360489	1.411582	1.4641	1.51807	1.573519
5	1.402552	1.469328	1.538624	1.61051	1.685058	1.762342
6	1.50073	1.586874	1.6771	1.771561	1.870414	1.973822
7	1.605781	1.713824	1.828039	1.948717	2.07616	2.210681
8	1.718186	1.85093	1.992563	2.143589	2.304537	2.475963
9	1.838459	1.999005	2.171893	2.357948	2.558036	2.773078
10	1.967151	2.158925	2.367364	2.593742	2.83942	3.105848
11	2.104852	2.331639	2.580426	2.853117	3.151757	3.478549
12	2.252192	2.51817	2.812665	3.138428	3.49845	3.895975
13	2.409845	2.719624	3.065805	3.452271	3.883279	4.363492
14	2.578534	2.937194	3.341727	3.797498	4.31044	4.887111
15	2.759031	3.172169	3.642482	4.177248	4.784588	5.473565
16	2.952164	3.425943	3.970306	4.594973	5.310893	6.130392
17	3.158815	3.700018	4.327633	5.05447	5.895091	6.86604
18	3.379932	3.996019	4.71712	5.559917	6.543551	7.689964
19	3.616527	4.315701	5.141661	6.115909	7.263342	8.61276
20	3.869684	4.660957	5.604411	6.7275	8.062309	9.646291

Scalene Triangles whose Areas and Sides are Integral

4	13	15	20	37	51
13	14	15	25	39	56
7	15	20	25	52	63
11	13	20	25	51	52
10	17	21	25	74	77
12	17	25	26	51	55
13	20	21	29	52	69
17	25	26	34	65	93
17	25	28	35	53	66
13	37	40	36	61	65
13	40	45	37	91	66
15	34	35	39	41	50
15	37	44	39	85	92
17	39	44	40	51	77
25	29	36	41	51	58
25	39	40	41	84	85
29	35	48	48	85	91
39	41	50	50	69	73
13	68	75	51	52	53
15	41	52	52	73	75
17	55	60	43	61	68

Right Triangles whose Sides are Integral

3	4	5	66	88	110
6	8	10	69	92	115
9	12	15	72	96	120
12	16	20	75	100	125
15	20	25	78	104	130
18	24	30	81	108	135
21	28	35	84	112	140
24	32	40	87	116	145
27	36	45	90	120	150
30	40	50	93	124	155
33	44	55	96	128	160
36	48	60	99	132	165
39	52	65	102	136	170
42	56	70	105	140	175
45	60	75	108	144	180
48	64	80	111	148	185
51	68	85	114	152	190
54	72	90	117	156	195
57	76	95	120	160	200
60	80	100	123	164	205
63	84	105	126	168	210

Squares of Integers from 10 to 100

No.	0	1	2	3	4	5	6	7	8	9
10	100	121	144	169	196	225	256	289	324	361
20	400	441	484	529	576	625	676	729	784	841
30	900	961	1024	1089	1156	1225	1296	1369	1444	1521
40	1600	1681	1764	1849	1936	2025	2116	2209	2304	2401
50	2500	2601	2704	2809	2916	3025	3136	3249	3364	3481
60	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
70	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
80	6400	6561	6724	6889	7056	7225	7396	7569	7744	7921
90	8100	8281	8464	8649	8836	9025	9216	9409	9604	9801

SQUARE ROOTS OF NUMBERS FROM 0 TO 10, AT INTERVALS OF .1

No.	.0	.1	.2	3	.4	.5	.6	.7	.8	.9
0	0	.316	.447	.548	.632	.707	.775	.837	.894	.949
1	1.000	1.049	1.095	1.140	1.183	1.225	1.265	1.304	1.342	1.378
2	1.414	1.449	1.483	1.517	1.549	1.581	1.612	1.643	1.673	1.703
3	1.732	1.761	1.789	1.817	1.844	1.871	1.897	1.924	1.949	1.975
4	2.000	2.025	2.049	2.074	2.098	2.121	2.145	2.168	2.191	2.214
5	2.236	2.258	2.280	2.302	2.324	2.345	2.366	2.387	2.408	2.429
6	2.449	2.470	2.490	2.510	2.530	2.550	2.569	2.588	2.608	2.627
7	2.646	2.665	2.683	2.702	2.720	2.739	2.757	2.775	2.793	2.811
8	2.828	2.846	2.864	2.881	2.898	2.915	2.933	2.950	2.966	2.983
9	3.000	3.017	3.033	3.050	3.066	3.082	3.098	3.114	3.130	3.146

SQUARE ROOTS OF INTEGERS FROM 10 TO 100

No.	0	1	2	3	4	5	6	7	8	9
10	3.162	3.317	3.464	3.606	3.742	3.873	4.000	4.123	4.243	4.359
20	4.472	4.583	4.690	4.796	4.899	5.000	5.099	5.196	5.292	5.385
30	5.477	5.568	5.657	5.745	5.831	5.916	6.000	6.083	6.164	6.245
40	6.325	6.403	6.481	6.557	6.633	6.708	6.782	6.856	6.928	7.000
50	7.071	7.141	7.211	7.280	7.348	7.416	7.483	7.550	7.616	7.681
60	7.746	7.810	7.874	7.937	8.000	8.062	8.124	8.185	8.246	8.307
70	8.367	8.426	8.485	8.544	8.602	8.660	8.718	8.775	8.832	8.888
80	8.944	9.000	9.055	9.110	9.165	9.220	9.274	9.327	9.381	9.434
90	9.487	9.539	9.592	9.644	9.695	9.747	9.798	9.849	9.899	9.950

CUBE ROOTS OF INTEGERS FROM 1 TO 30

No.	CUBE ROOT	No.	CUBE ROOT	No.	CUBE ROOT
1	1.000000	11	2.223980	21	2.758924
2	1.259921	12	2.289429	22	2.802039
3	1.442250	13	2.351335	23	2.843867
4	1.587401	14	2.410142	24	2.884499
5	1.709976	15	2.466212	25	2.924018
6	1.817121	16	2.519842	26	2.962496
7	1.912931	17	2.571282	27	3.000000
8	2.000000	18	2.620741	28	3.036589
9	2.080084	19	2.668402	29	3.072317
10	2 154435	20	2.714418	30	3.107233

TABLES OF PRIME NUMBERS FROM 1 TO 1000

1	109	269	439	617	811
2	13	71	43	19	21
3	27	77	49	31	23
5	31	81	57	41	27
7	37	83	61	43	29
11	39	93	63	47	39
13	49	307	67	53	53
17	51	11	79	59	57
19	57	13	87	61	59
23	63	17	91	73	63
29	67	31	99	77	77
31	73	37	503	83	81
37	79	47	09	91	83
41	81	49	21	701	87
43	91	53	23	09	907
47	93	59	41	19	11
53	97	67	47	27	19
59	99	73	57	33	29
61	211	79	63	39	37
67	23	83	69	43	41
71	27	89	71	51	47
73	29	97	77	57	53
79	33	401	87	61	67
83	39	09	93	69	71
89	41	19	99	73	77
97	51	21	601	87	83
101	57	31	07	97	91
03	63	33	13	809	97
07					

NOTE.—The hundreds' digits are not repeated after being first introduced, unless at the heads of columns.

CONSTANTS

$$\pi = 3.14159265359$$

$$\frac{\pi}{4} = 0.7853982$$

$$\frac{\pi}{6} = 0.5235988$$

$$\frac{1}{\pi} = 0.3183099$$

$$\pi^2 = 9.8696044$$

$$\frac{1}{\pi^2} = 0.1013212$$

$$\sqrt{\pi} = 1.7724539$$

$$\frac{1}{\sqrt{\pi}} = 0.5641896$$

$$\log \pi = 0.4971499$$

$$\log \frac{\pi}{4} = 9.8950899 - 10$$

$$\log \frac{\pi}{6} = 9.7189986 - 10$$

$$\log \frac{1}{\pi} = 9.5028501 - 10$$

$$\log \pi^2 = 0.9942997$$

$$\log \frac{1}{\pi^2} = 9.0057003 - 10$$

$$\log \sqrt{\pi} = 0.2485749$$

$$\log \frac{1}{\sqrt{\pi}} = 9.7514251 - 10$$

Specific Gravities. — Water 1

A table showing the weight of each substance compared with an equal volume of pure water. A cubic foot of rain water weighs 1000 ounces, or 62½ lb. Avoir. To find the weight of a *cubic foot* of any substance named in the table, move the decimal point three places toward the right, which is multiplying by 1000, and the result will show the number of ounces in a cubic foot.

Substances	Specific Grav.	Substances	Specific Grav.
Acid, acetic	1.008	Lead, cast	11.350
Acid, nitric	1.271	Lead, white	7.235
Acid, sulphuric	1.841 to 2.125	Lead, ore	7.250
Air	.001227	Lignum vitæ	1.333
Alcohol, of commerce	.835	Lime	.804
Alcohol, pure	.794	Lime, stone	2.386
Alder wood	.800	Mahogany	1.063
Ale	1.035	Manganese	3.700
Alum	1.724	Maple	.750
Aluminum	2.560	Marble	2.716
Amber	1.064	Men (living)	.891
Amethyst	2.750	Mercury, pure	14.000
Ammonia	.875	Mica	2.750
Ash	8.400	Milk	1.032
Blood, human	1.054	Nickel	8.279
Brass (about)	8.400	Niter	1.900
Brick	2.000	Oil, castor	.970
Butter	.942	Oil, linseed	.940
Cedar	.457 to .561	Opal	2.114
Cherry	.715	Opium	1.337
Cider	1.018	Pearl	2.510
Coal, bituminous (about)	1.250	Pewter	7.471
Coal, anthracite	1.500	Platinum (native)	17.000
Copper	8.788	Platinum, wire	21.041
Coral	2.540	Poplar	.383
Cork	.240	Porcelain	2.385
Diamond	3.530	Quartz	2.500
Earth (mean of the globe)	5.210	Rosin	1.100
Elm	.661	Salt	2.130
Emerald	2.678	Sand	1.500 to 1.800
Fir	.550	Silver, cast	10.474
Glass, flint	2.760	Silver, coin	10.534
Glass, plate	2.760	Slate	2.110
Gold, native	15.600 to 19.500	Steel	7.816
Gold, pure, cast	19.258	Stone	2.000 to 2.700
Gold, coin	17.647	Sulphur, fused	1.990
Granite	2.652	Tallow	.941
Gum Arabic	1.452	Tar	1.015
Gypsum	2.288	Tin	7.291
Honey	1.456	Turpentine, spirits of	.870
Ice	.930	Vinegar	1.013
Iodine	4.948	Walnut	.671
Iron	7.645	Water, distilled	1.000
Iron, ore	4.900	Water, sea	1.028
Ivory	1.917	Wax	.897
Lard	.947	Zinc, cast	7.190

Approximate Values of Foreign Coins in United States Money

Country	Standard	Monetary Unit	Value in Terms of U. S. Gold Dollars
Argentine Republic	Gold & Silver	Peso	.965
Austria-Hungary	Gold	Crown	.203
Belgium	Gold & Silver	Franc	.193
Bolivia	Silver	Boliviano	.441
Brazil	Gold	Milreis	.546
British Possessions in N. A. [*except Newfoundland*]	Gold	Dollar	1.00
Central Am. States Guatemala Honduras Nicaragua Salvador	Silver	Peso	.441
Chili	Gold	Peso	.365
China	Silver	Tael Shanghai	.661
		Haikwan	.736
		Canton	.722
Colombia	Gold	Dollar	1.00
Costa Rica	Gold	Colon	.465
Cuba	Gold	Peso	.91
Denmark	Gold	Crown	.268
Ecuador	Gold	Sucre	.487
Egypt	Gold	Pound [100 Piastres]	4.943
Finland	Gold	Mark	.193
France	Gold & Silver	Franc	.193
German Empire	Gold	Mark	.238
Great Britain	Gold	Pound Sterling	4.866½
Greece	Gold & Silver	Drachma	.193
Haiti	Gold & Silver	Gourde	.965
India	Gold	Pound Sterling	4.866½
Italy	Gold & Silver	Lira	.193
Japan	Gold	Yen, Gold	.498
Liberia	Gold	Dollar	1.00
Mexico	Gold	Peso	.498
Netherlands	Gold & Silver	Florin	.402
Newfoundland	Gold	Dollar	1.014
Norway	Gold	Crown	.268
Peru	Gold	Sol	.487
Portugal	Gold	Milreis	1.08
Russia	Gold	Rouble, Gold	.515
Spain	Gold & Silver	Peseta	.193
Sweden	Gold	Crown	.268
Switzerland	Gold & Silver	Franc	.193
Tripoli	Silver	Mahbub [20 Piastres]	.413
Turkey	Gold	Piastre	.044
Venezuela	Gold & Silver	Bolivar	.193

WEIGHTS AND MEASURES

AVOIRDUPOIS WEIGHT

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight, or 2000 pounds = 1 ton (T.)

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1 ton = 20 cwt. = 2000 lb. = 32,000 oz.
1 pound Avoirdupois weight = 7000 grains.
1 ounce Avoirdupois weight = 437½ gr.

TROY WEIGHT

24 grains (gr.) = 1 pennyweight (pwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

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1 lb. = 12 oz. = 240 pwt. = 5760 gr.
1 ounce Troy weight = 480 gr.

APOTHECARIES' WEIGHT

20 grains (gr. xx) = 1 scruple (sc., or ℈)
3 scruples (℈iij) = 1 dram (dr., or ʒ)
8 drams (ʒviij) = 1 ounce (oz., or ℥)
12 ounces (℥xij) = 1 pound (lb., or ℔.)

---

1 ℔. = 12 ℥ = 96 ʒ = 288 ℈ = 5760 gr.
Medicines are bought and sold in quantities by Avoirdupois weight.

APOTHECARIES' FLUID MEASURE

60 minims, or drops (♏, or gtt.) = 1 fluidrachm (fʒ)
8 fluidrachms = 1 fluidounce (f℥)
16 fluidounces = 1 pint (O.)
8 pints = 1 gallon (Cong.)

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1 Cong. = 8 O. = 128 f℥ = 1024 fʒ = 61,440 ♏.
O. is an abbreviation of octans, the Latin for one eighth; Cong. for congiarium, the Latin for gallon.

LINEAR MEASURE

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods = 1 mile (mi.)

1 mi. = 320 rd. = 1760 yd. = 5280 ft. = 63,360 in.

MARINERS' LINEAR MEASURE

9 inches (in.) = 1 span (sp.)
8 spans, or 6 feet = 1 fathom (fath.)
120 fathoms = 1 cable's length (c. l.)
$7\frac{1}{3}$ cable lengths = 1 nautical mile (or knot) (mi.)
3 miles = 1 league

GEOGRAPHICAL AND ASTRONOMICAL LINEAR MEASURE

1 geographic mile = 1.15 statute miles
3 geographic miles = 1 league
60 geographic miles, or 69.16 statute miles = 1 degree of latitude on a meridian, or of longitude on the equator
360 degrees = the circumference of the earth

SURVEYOR'S LINEAR MEASURE

7.92 inches = 1 link (l.)
25 links = 1 rod (rd.)
4 rods = 1 chain (ch.)
80 chains = 1 mile (mi.)

1 mile = 80 ch. = 320 rd. = 8000 l. = 63,360 in.

JEWISH LINEAR MEASURE

cubit = 1.824 ft.
Sabbath day's journey = 3648 ft.
mile (4000 cubits) = 7296 ft.
day's journey = 33.164 mi.

SQUARE MEASURE

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards = 1 square rod or perch (sq. rd.; P.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

sq. mi.		A.		sq. rd.		sq. yd.		sq. ft.		sq. in.
1	=	640	=	102,400	=	3,097,600	=	27,878,400	=	4,014,489,600
		1	=	160	=	4840	=	43,560	=	6,272,640
				1	=	30¼	=	272¼	=	39,204
						1	=	9	=	1296

Surveyor's Square Measure

625 square links (sq. l.) = 1 square rod (sq. rd.)
16 square rods = 1 square chain (sq. ch.)
10 square chains = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
36 square miles = 1 township (Tp.)

Tp.		sq. mi.		A.		sq. ch.		sq. rd.		sq. l.
1	=	36	=	23,040	=	230,400	=	3,686,400	=	2,304,000,000
		1	=	640	=	6400	=	102,400	=	6,400,000
				1	=	10	=	160	=	100,000

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

1 cu. yd. = 27 cu. ft. = 46,656 cu. in.

Wood Measure

16 cubic feet = 1 cord foot (cd. ft.)
8 cord feet, or 128 cubic feet } = 1 cord (cd.)
24¾ cubic feet = { perch (Pch.) of stone or of masonry

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

1 bu. = 4 pk. = 32 qt. = 64 pt.

Liquid Measure

4 gills = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
31½ gallons = 1 barrel (bbl.)

1 bbl. = 31½ gal. = 126 qt. = 252 pt. = 1008 gi.

CIRCULAR MEASURE

60 seconds = 1 minute (′)
60 minutes = 1 degree (°)
360 degrees = 1 circle

COMMERCIAL WEIGHT

16 drams = 1 ounce (oz.)
16 ounces = 1 pound (lb.)
2000 pounds = 1 ton (T.)

PAPER

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

ENGLISH MONEY

4 farthings (far.) = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound (£)

1 £ = 20*s.* = 240*d.* = 960 far.
1 £ = $4.8665 in U. S. money

MEASURE OF TIME

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days = 1 year (yr.)
366 days = 1 leap year
1 da. = 24 hr. = 1440 min. = 86,400 sec.

THE METRIC SYSTEM

(*The Acme of Simplicity*)

The following prefixes are used in the Metric System:

(GREEK)		(LATIN)	
deka, meaning	10	deci, meaning	.1
hekto, meaning	100	centi, meaning	.01
kilo, meaning	1000	milli, meaning	.001
myria, meaning	10,000		

Linear Measure

10 millimeters (mm.) = 1 centimeter (cm.)
10 centimeters = 1 decimeter (dm.)
10 decimeters = 1 meter (m.)
10 meters = 1 dekameter (Dm.)
10 dekameters = 1 hektometer (Hm.)
10 hektometers = 1 kilometer (Km.)
10 kilometers = 1 myriameter (Mm.)

Square Measure

100 square millimeters (sq. mm.) = 1 square centimeter (sq. cm.)
100 square centimeters = 1 square decimeter (sq. dm.)
100 square decimeters = 1 square meter (sq. m.)
100 square meters = 1 square dekameter (sq. Dm.)
100 square dekameters = 1 square hektometer (sq. Hm.)
100 square hektometers = 1 square kilometer (sq. Km.)

The area of a farm is expressed in hektares.
The area of a country is expressed in square kilometers.

Cubic Measure

1000 cubic millimeters (cu. mm.) = 1 cubic centimeter (cu. cm.)
1000 cubic centimeters = 1 cubic decimeter (cu. dm.)
1000 cubic decimeters = 1 cubic meter (cu. m.)

Table of Capacity

10 milliliters (ml.) = 1 centiliter (cl.)
10 centiliters = 1 deciliter (dl.)
10 deciliters = 1 liter (l.)
10 liters = 1 dekaliter (Dl.)
10 dekaliters = 1 hektoliter (Hl.)
10 hektoliters = 1 kiloliter (Kl.)
10 kiloliters = 1 myrialiter (Ml.)

The hektoliter is used in measuring grain, vegetables, etc.
The liter is used in measuring liquids and small fruits.

Table of Weight

10 milligrams (mg.) = 1 centigram (cg.)
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 gram (g.)

10 grams = 1 dekagram (Dg.)
10 dekagrams = 1 hektogram (Hg.)
10 hektograms = 1 kilogram (Kg.)
1000 kilograms = 1 metric ton (T.)

A myriagram = 10,000 grams
A quintal (Q.) = 100,000 grams

TABLE OF EQUIVALENTS

Long Measure

1 inch = 2.54 centimeters
1 foot = .3048 of a meter
1 yard = .9144 of a meter
1 rod = 5.029 meters
1 mile = 1.6093 kilometers

1 centimeter = .3937 of an inch
1 decimeter = .328 of a foot
1 meter = 1.0936 yards
1 dekameter = 1.9884 rods
1 kilometer = .62137 of a mile

Square Measure

1 square inch = 6.452 square centimeters
1 square foot = .0929 of a square meter
1 square yard = .8361 of a square meter
1 square rod = 25.293 square meters
1 acre = 40.47 ares
1 square mile = 259 hectares

1 square centimeter = .155 of a square inch
1 square decimeter = .1076 of a square foot
1 square meter = 1.196 square yards
1 are = 3.954 square rods
1 hektare = 2.471 acres
1 square kilometer = .3861 of a square mile

Cubic Measure

1 cubic inch = 16.387 cubic centimeters
1 cubic foot = 28.317 cubic decimeters
1 cubic yard = .7645 of a cubic meter
1 cord = 3.624 steres

1 cubic centimeter = .061 of a cubic inch
1 cubic decimeter = .0353 of a cubic foot
1 cubic meter = 1.308 cubic yards
1 stere = .2759 of a cord

Measures of Capacity

1 liquid quart = .9463 of a liter
1 dry quart = 1.101 liters
1 liquid gallon = .3785 of a dekaliter
1 peck = .881 of a dekaliter
1 bushel = .3524 of a hektoliter

1 liter = 1.0567 liquid quarts
1 liter = .908 of a dry quart
1 dekaliter = 2.6417 liquid gallons
1 dekaliter = 1.135 pecks
1 hektoliter = 2.8375 bushels

Measures of Weight

1 grain Troy = .0648 of a gram
1 ounce Avoirdupois = 28.35 grams
1 ounce Troy = 31.104 grams
1 pound Avoirdupois = .4536 of a kilogram
1 pound Troy = .3732 of a kilogram
1 ton (short) = .9072 of a tonneau

1 gram = .03527 of an ounce Avoirdupois
1 gram = .03215 of an ounce Troy
1 gram = 15.432 grains Troy
1 kilogram = 2.2046 pounds Avoirdupois
1 kilogram = 2.679 pounds Troy
1 tonneau = 1.1023 tons (short)

CONVENIENT MULTIPLES FOR CONVERSION

To Convert		
Grains to Grams,	multiply by	.065
Ounces to Grams,	multiply by	28.35
Pounds to Grams,	multiply by	453.6
Pounds to Kilograms,	multiply by	.45
Hundredweights to Kilograms,	multiply by	50.8
Tons to Kilograms,	multiply by	1016.
Grams to Grains,	multiply by	15.4
Grams to Ounces,	multiply by	.35
Kilograms to Ounces,	multiply by	35.3
Kilograms to Pounds,	multiply by	2.2
Kilograms to Hundredweights,	multiply by	.02

Kilograms to Tons,	multiply by	.001
Inches to Millimeters,	multiply by	25.4
Inches to Centimeters,	multiply by	2.54
Feet to Meters,	multiply by	.3048
Yards to Meters,	multiply by	.9144
Yards to Kilometers,	multiply by	.0009
Miles to Kilometers,	multiply by	1.6
Millimeters to Inches,	multiply by	.04
Centimeters to Inches,	multiply by	.4
Meters to Feet,	multiply by	3.3
Meters to Yards,	multiply by	1.1
Kilometers to Yards,	multiply by	1093.6
Kilometers to Miles,	multiply by	.62

MISCELLANEOUS

Acre = 5645.376 square varas.

Acre (square) is $209\frac{2}{3}$ feet each way.

Ampere (unit of current) is that current of electricity that decomposes .00009324 gram of water per second.

Are = a square dekameter.

Barleycorn = $\frac{1}{3}$ inch.

Barrel (flour) weighs 196 pounds.

Barrel (wine) holds 31 gallons.

Bushel (imperial) = 2216.192 cubic inches.

Bushel (U. S.) = 2150.42 cubic inches.

Cable length = 120 fathoms.

Calorie = 42,000,000 ergs = .428 kilogrammeter.

Carat (assayer's weight) = 10 pennyweight.

Carat (of diamond) = $3\frac{1}{5}$ grains.

Centare = 1 square meter.

Century = 100 years.

Chaldron = 36 bushels.

Coulomb (unit of quantity) is a current of 1 ampere during 1 second of time.

Crown = 5 shillings.

Cubic foot of water weighs $62\frac{1}{2}$ pounds.

Cubit = 18 inches.

Cycle (metonic) = 19 years.

Cycle (of indiction) = 15 years.

Cycle (solar) = 28 years.

Degree (1°) = $\frac{1}{90}$ of a right angle = $\frac{\pi}{180}$ radian.

Dozen = 12.

Dozen (baker's) = 13.

Eagle = $10.

Farthing = $ .00503.

Fathom = 6 feet.

Firkin (wine measure) = 9 gallons.

Florin (Austrian) = $4.53.

Fortnight = 2 weeks.

Furlong = $\frac{1}{8}$ mile.

Gallon (dry) = 268 cubic inches.

Gallon (liquid) = 231 cubic inches.

Gram = weight of 1 cubic centimeter of distilled water at its maximum density.

Great gross = 12 gross.

Gross = 12 dozen.

Gross ton, long ton = 2240 pounds.

Guilder (Holland) = $ .402.

Guinea = 21 shillings.

Half section = 320 acres.

Hand = 4 inches.

Heat of fusion of ice at 0° C. is 80 calories per gram.

Heat of vaporization of water at 100° C. is 536 calories per gram.

Hectare = 1 square hectometer.

Hogshead = 2 barrels.

Kilo = a kilogram.

Knot = 6086 feet, or 1.15 miles.

Labor = 177.136 acres.

League or Sitio (Spanish) = 4428.4 acres.

Leap year. The centennial years divisible by 400 and all other years divisible by 4 are leap years.

Light travels 300,000,000 meters, or 186,000 miles, per second.

Liter = 1 cubic decimeter.

Long hundredweight = 112 pounds.

Mill = $ .001.

Minim = a drop of pure water.

Mite = $ .0187.

Nautical mile = 1 knot.

Ohm (unit of resistance) is the electrical resistance of a column of mercury 106 centimeters long and of 1 square millimeter section.

Pace (common) = 3 feet.
Pace (military) = $2\frac{1}{2}$ feet.
Pack = 240 pounds.
Parcian (Spanish) = 5314.08 acres.
Penny = $.02025.
Period (Dionysian, or Paschal) = 532 years.
Quarter (English) = 8 bushels; U. S. = $8\frac{1}{2}$ bushels.
Quarter section = 160 acres.
Quintal = 100,000 grams.
Radian = $\frac{180}{\pi} = 57.2957796°$.
Rod = $5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet.
Score = 20.
Section = 640 acres.
Sextant = 60°.
Shilling = $ .243.
Sign = 30 degrees.
Span = 9 inches.
Specific heat of ice is about 0.505.
Square = 100 square feet.
Stere = .2759 cord, or 1 cubic meter.
Stone = 14 pounds.
Strike (dry measure) = 2 bushels.
Ton (long) = 2240 pounds.
Ton (register) = 100 cubic feet.
Ton (shipping) = 40 cubic feet.
Ton (short) = 2000 pounds.
Tonneau = 1.1023 tons.
Township = 36 square miles.
Vara (California) = 33 inches.
Vara (Texas) = .9259 + yard, or $33\frac{1}{3}$ inches.

Volt (unit of electromotive force) is 1 ampere of current passing through a substance having 1 ohm of resistance.

Watt (unit of power) is the power of 1 ampere current passing through a resistance of 1 ohm.

Year (common) = 365 days.
Year (leap, or bissextile) = 366 days.
Year (lunar) = 354 days.
Year (sidereal) = 365 days, 6 hours, 9 minutes, 9 seconds.
Year (solar) = 365 days, 5 hours, 48 minutes, 46.05 seconds.

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